Report to the Ministry of Education

Mathematics and statistics skills and knowledge learners need to know by when, important cross-disciplinary links, and considerations in light of rapid changes and growth in computer science/ICT

September 2021

Prepared by Robin Averill, Fiona Ell, and Jane McChesney

Contents

Report focus areas

The focus areas requested for this report were:

- The mathematics and statistics skills and knowledge learners need to know by when
- Important cross-disciplinary links between mathematics, statistics and other discipline areas
- Considerations regarding rapid changes and growth in computer science/ICT

Overview of the report

In the report we describe the ways we worked, the national context, recommendations in light of the signalled bicultural nature of the curriculum refresh, rationale for the organisation of mathematics content and key recommendations for organisation of mathematics and statistics content across the year levels, explanation of the template used to show year-by-year mathematics and statistics focus points, discussion of important cross-discipline links for mathematics and statistics, considerations regarding rapid changes and growth in computer science, consideration regarding other aspects of pedagogy, and final comments. Year-by-year indications of mathematics and statistics focus points, along with suggested overview purposes of each part of the mathematics and statistics curriculum are included.

Process

The task was explained to the team in a one-day hui at the Ministry of Education on 13 August 2021, followed by the team planning an approach for working together and sharing tasks. Collaboration continued over the subsequent two weeks with frequent email and Zoom discussions to clarify process matters and agreed responses to the tasks set. Our main reference points for developing the report have been the current and previous iterations of the New Zealand mathematics and statistics curriculum, international examples, literature, and our varied experiences locally, nationally and internationally within mathematics and statistics education. Consultation possibilities were constrained by the timeframe given for the preparation of the report and the country being in level 4 lockdown over this time. However, we are very grateful to Dr Karyn Saunders, Dr Pip Arnold and Anna Fergusson for their very kind and swift feedback on selected ideas within the year-by-year suggested content.

National context of mathematics and statistics education

The report was completed within a time of substantial change and consultation across a wide range of general and mathematics and statistics-specific national education developments, such as the NCEA review, development and piloting of numeracy NCEA co-requisites, the curriculum refresh, and a Royal Society expert panel report on mathematics and statistics education. The report team have had varying involvements in these developments but carried out the work for this report independent of but with some consideration towards these other initiatives and developments. Furthermore, *Tātaiako* (Education Council, 2011) and *Tapasā* (Ministry of Education, 2018) have been published and advocated for teacher use since the 2007 national curriculum was written, and the mathematics and statistics curriculum has not yet been revisited in light of these significant documents and related research, professional development and understandings in the area of culturally sustaining practice.

Mathematics and statistics in a bicultural curriculum

The curriculum refresh is intending a bicultural curriculum[1]. This report was completed without guidance on what it is intended for a bicultural mathematics and statistics curriculum. The starting points for our work are Euro-centric rather than bicultural by design. Since the 2007 New Zealand curriculum came into effect, kaiako have been expected to respond in their teaching to *Tātaiako* and a new professional code and standards (Education Council, 2017), both of which enhance the place of te reo me te ao Māori in their work, but are not explicit about what this means for the teaching of mathematics and statistics. We have tried to include some emphasis consistent with a bicultural curriculum. However, we are writing from the position of Tangata Tiriti, not Tangata Whenua. Tangata Whenua voices are clearly essential to determine what mathematics and statistics are needed by when, cross-curriculum links, and the impacts of rapid ICT growth, for a bicultural mathematics and statistics curriculum. The basic concept of what 'progress' might be is itself a cultural construct. For the curriculum to be bicultural it should have bicultural origins and originating frameworks which we cannot provide. Control over what is in the mathematics and statistics curriculum should be fundamentally shared. Our contribution is made to help progress the Ministry of Education's work, but it needs to be considered in light of this fact.

Recommendations:

- We strongly advocate for further work to discuss and describe what is meant by a bicultural mathematics and statistics curriculum, including what mātauranga Māori and mātaraunga-a-iwi is included when and how it is included, before writing of the refreshed mathematics and statistics curriculum begins.

[1] https://www.education.govt.nz/our-work/changes-in-education/national-curriculum-refresh/new-zealand-curriculum/

We recommend further work occurs to:

- reconsider what mathematics and statistics are needed by ākonga by when in light of the recommendation above
- consider further ways in which the curriculum will be written, portrayed and implemented as a bicultural curriculum (e.g., considering protocols, pedagogies, cultural competencies, te reo Māori, learning environment contexts, contexts of learning activities, awareness of how people have been involved in the development and use of mathematical and statistical ideas and tools over time, involvement of whānau and families in supporting learning, developing and nurturing culturally-linked values)
- discuss, decide, and describe ways to support kaiako in designing bicultural mathematics and statistics learning opportunities for ākonga
- consider consultation and communication with whānau, families, and society needed for supporting curriculum implementation.

Other considerations regarding culturally sustaining mathematics and statistics

Alongside consideration of mātauranga Māori and mātauranga-a-iwi within mathematics and statistics content, it is vital to consider responsibilities to our learners with Pacific heritage in relation to curriculum content and emphasis. Links between *Tātaiako* and *Tapasā* and mathematics and statistics must be made explicit by curriculum writers. We note and are supportive of the framing of mathematics and statistics and links to the important ideas of these documents in the draft NCEA numeracy corequisite materials[2], as in the 'Numeracy Learning Matrix' and the 'Unpacking Numeracy' document (p. 1) and in the table showing links between mathematical process ideas, the key competencies, *Tātaikao* and *Tapasā* (final page of Unpacking document). The table makes links between mathematical processes and these competencies.

Recommendation:

- We strongly advocate consideration of mathematics and statistics content in light of *Tātaiako* and *Tapasā* and related knowledge and understanding about the groups

[2] https://ncea.education.govt.nz/numeracy-0 and
Australian Curriculum, Assessment and Reporting Authority. (2021). *Mathematics consultation curriculum, All elements F-10.*
https://ncea-live-3-storagestack-53q-assetstorages3bucket-2o21xte0r81u.s3.amazonaws.com/s3fs-public/2021-08/Numeracy%20Learning%20Matrix%20%28A4%29.pdf?VersionId=Nw0dtOQPEF6bNm8Z6YooJBYasvB47udf
https://ncea-live-3-storagestack-53q-assetstorages3bucket-2o21xte0r81u.s3.amazonaws.com/s3fs-public/2021-08/Unpacking%20Numeracy%20%28A4%3B%20Page%206%20is%20A3%29.pdf?VersionId=j.cT_3zjOJWnrwHFdOe83PbIKltnDj4b

these documents are seeking to serve. Further, these links need to be made explicit and visible in curriculum materials to help teachers see and use them.

The mathematics skills and knowledge learners need to know by when: Years 0-13

We need to be specific about the mathematics and statistics that we want ākonga to learn each year they are at school because wider bands (e.g., such as the current curriculum levels, or considering group of years such Years 0-3, 4-6 etc) make it hard for teachers to know what they should be aiming for and whether ākonga have made appropriate progress for further success. Making local curriculum is a complex task for teachers and clear guidelines will assist.

Year-by-year structure

Curriculum documents serve a number of purposes in education systems. By their structure, as well as their content, they convey important messages about what is valued and how knowledge, skills and attitudes are perceived. Some purposes for curriculum are well-served by high level, broad and open documents, which provide space for local interpretation. Other purposes are well-served by providing more specific information that provides clear direction. Both approaches can be used together within a curriculum to serve both sets of purposes.

In mathematics and statistics education in English medium schools in Aotearoa we have three key issues made evident considering international comparative studies:

- There is dissatisfaction with the achievement profile and rate of progress of ākonga. Ākonga are not learning enough mathematics and statistics, and they are not learning it soon enough.
- There are major equity problems in mathematics and statistics achievement and progress, and engagement and participation. There are larger gaps between those who do well and those who do not than in other countries, Māori and Pacific learners are not getting sufficient access to learning experiences that enable them to achieve in the same way as Pākehā and Asian ākonga, and SES has a bigger impact on achievement and progress than in other jurisdictions, with schooling making less of an impact on the effects of poverty than in other, comparable, places[3].
- Variability in ākonga outcomes on PISA tests is attributed to 'within school' factors more than 'between school' factors in Aotearoa. Our 'within school' variability is amongst the highest in the PISA jurisdictions[4]. NMSSA data also shows that ākonga

[3] Ministry of Education. (2020). https://www.educationcounts.govt.nz/__data/assets/pdf_file/0015/205710/TIMSS-2018-Maths-Achievement-A4.pdf
[4] OECD. (2019). https://www.oecd.org/pisa/PISA%202018%20Insights%20and%20Interpretations%20FINAL%20PDF.pdf

outcomes in mathematics and statistics are shaped by school decile[5]. Reducing within-school and between-school variability is a key to improving ākonga outcomes.

With the opportunity to refresh the mathematics and statistics curriculum we need to ask: what sort of curriculum structure will best address these key issues? While the curriculum structure is not the most immediate influence on teaching and learning, it provides a crucial framework for how teachers, ākonga and communities think about mathematics and statistics teaching and learning[6]. The structure needs to be the one most likely to result in:

- Sufficient progress and achievement to meet our goals
- Equity in access and in outcomes
- Reduction in variability of learning experiences and achievement opportunities

There is very little (no) direct research on how curriculum structure impacts system outcomes. There is some work on how teachers work with materials to design learning, but there is not a large body of evidence about how this process occurs between national curriculum statements and classroom programme design and implementation[7]. What evidence there is suggests that teachers can learn about both mathematics and statistics content and how it should be staged or approached through interacting with curriculum documents[8].

To decide what curriculum structure might best promote achievement, progress, equity and consistency across the system we therefore have to turn to thinking about what it is that teachers and leaders need in order to provide opportunities for learning mathematics and statistics that will achieve these aims. Teacher knowledge of mathematics for teaching is recognised as a significant factor in quality of mathematics teaching[9]. One way to support teacher knowledge of mathematics for teaching is to provide a curriculum that clarifies for teachers what they should be teaching. In presenting clear curriculum direction and specificity, teachers have access to

[5] NMSSA. (2018). https://nmssa.otago.ac.nz/reports/2018/2018_NMSSA_MATHEMATICS.pdf

[6] Schmidt, W. H., & Prawat, R. S. (2006). Curriculum coherence and national control of education: Issue or non-issue? *Journal of Curriculum Studies, 38*(6), 641–658.

[7] Remillard, J. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211-246.
Sullivan, P., Clarke, D., Clarke, D., Farrell, L., & Gerrard, J. (2013). Processes and principles in planning mathematics teaching. *Mathematics Education Research Journal, 24*(4), 457-480.

[8] Choppin, J. (2011). Learned adaptations: Teachers' understanding and use of curriculum resources. *Journal of Mathematics Teacher Education, 14*, 331-353.
Remillard, J. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211-246.

[9] Ball,D., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59* (5), 389-407.
Charalambous, C. Hill, H., Chin, M., & McGinn,D. (2020). Mathematical content knowledge and knowledge for teaching: exploring their distinguishability and contribution to student learning. *Journal of Mathematics Teacher Education, 23*, 579-613.
Depaepe, F., Verschaffel, L., & Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education, 34*, 12-24.
Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: the knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*, 255-281.

information about mathematical content at a level of detail that expands/strengthens their knowledge bases for learning[10]. In particular, a curriculum can draw together evidence from developmental psychology, neuroscience, mathematics education, statistics education, mathematics and statistics disciplines, and other national priority areas to form outcomes for learners that highlight:

- Sequences for learning
- What is important for progress and future learning
- Connections among different ideas and processes

A curriculum should be a tool for teachers. It should help them be better at preparing for and teaching mathematics and statistics. The tool that a curriculum is most like is a map. The curriculum represents the 'terrain' of mathematics and statistics: the more detail that is included, the less likely you are to get lost. A map points out the important features of the landscape and shows you how to get there, and it shows how the different parts of the landscape relate to each other. Research on learning trajectories provides a strong base for mapping learning pathways in mathematics and statistics[11]. Some of this work suggests that setting out curriculum expectations for each year provides a number of advantages over broader bands of year levels. One advantage is in terms of clearly identifying learning sequence, particularly when there is an element of learning needed for later topics (requisite knowing). For some elements of content, learning at later times (either within a year or in following years) provides opportunities to revisit and consolidate content (reaching into prior learning), and can extend or connect these important elements[12]. These features of year-by-year curriculum illustrate the benefits of mapping for coherence across and within mathematical content topics. When the curriculum does not provide enough information for teachers to find their way, they turn to other 'maps' to help them: in our case, examples include the Numeracy Project framework, the National Standards, the Learning Progression Framework, commercial textbooks, online mathematics resource collections (e.g., Achievement Objective 'elaborations') and online programmes[13]. It is timely to return to more balance where the level of detail in the curriculum provides a greater shared common curriculum space for all schools and teachers (a curriculum mathematics and statistics 'commons'), while still enabling scope for adapting and designing for localised school contexts[14].

---

[10] Cobb, P., & Jackson, K. (2011). Assessing the quality of the Common Core State Standards for mathematics. *Educational Researcher, 40*(4), 183–185.

[11] e.g., Confrey, J., Maloney, A. P., & Corley, A. K. (2014). Learning trajectories: A framework for connecting standards with curriculum. *ZDM—The International Journal on Mathematics Education, 46*(5), 719–733. and Nguyen, K. H., & Confrey, J. (2014). Exploring the relationship between learning trajectories and curriculum. In A. P. Maloney, J. Confrey, & K. H. Nguyen (Eds.), *Learning over time: Learning trajectories in mathematics education* (pp. 161-185). Information Age Publishing.

[12] Rowland, T. (2014). Mathematics teacher knowledge. In P. Andrews & T. Rowland (Eds.), *Masterclass in Mathematics Education: International perspectives on teaching and learning* (pp. 87-98). Bloomsbury Academic.

[13] McChesney, J. (2017). Searching the New Zealand curriculum landscape for clarity and coherence: Some tensions in Mathematics and Statistics. *Curriculum Matters, 13*, 115-131.

[14] Prawat, R. S., & Schmidt, W. H. (2006), Curriculum coherence: Does the logic underlying the organisation of subject matter matter? In S. J. Howie & T. Plomp (Eds.), *Contexts of learning mathematics and science: Lessons learned from TIMSS* (pp. 265-276). Routledge.

In short, the 2007 NZC Mathematics and Statistics document does not have enough detail for teachers to plan Mathematics and Statistics programmes, and so they turn to other sources that give them enough detail, some created specifically for this purpose. This reaching outside the curriculum can be seen as a constructive process of teachers and schools taking local ownership of curriculum and meeting learner needs, but in practice teachers' knowledge of mathematics and statistics and the pressure experienced in teaching the full breadth of the curriculum means that it is more likely that they will adopt a proxy curriculum wholesale than they will craft a local programme. Mathematics and Statistics are not easy to teach, and there is a huge volume of information about how ākonga learn mathematics and statistics that teachers can never hope to master in the face of their other tasks. A more specific curriculum than the current 2007 curriculum is an opportunity to help teachers design and teach mathematics and statistics learning experiences that make a difference for all learners. Access to specific curriculum information, and opportunities to work together with colleagues, act as feedback loops for teachers who in turn consolidate their curricular and content knowledge for teaching[15].

Evidence from four contributing bodies of research can be used to make the decision to have a year-by-year curriculum rather than broader bands:

1. Early mathematics learning in primary school is very significant to future progress and has many small but important steps that could be missed if year-by-year detail is not provided.

There is a strong body of evidence that what ākonga learn early in their primary school mathematics journey is highly important for and predictive of their rate of progress and their achievement[16]. Particular concepts and skills have been highlighted that can be specified in a curriculum document and become part of everyday mathematics programmes[17]. The detail we have from research into learning trajectories[18] can be used to help teachers teach what is important and what will help all learners make good progress if it is included in the curriculum. There is also evidence that concepts in number and spatial relationships are 'tied together',

[15] Choppin, J. (2011). Learned adaptations: Teachers' understanding and use of curriculum resources. *Journal of Mathematics Teacher Education, 14*, 331-353.

[16] Mulligan, J. & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21* (2), 33-49.

Rittle-Johnson, B., Fyfe, E., Hofer, K., & Farran, D. (2017). Early math trajectories: Low-income children's mathematics knowledge from ages 4 to 11. *Child Development, 88* (5), 1727-1742.

Wilkins, J., Woodward, D., & Norton, A. (2021). Children's number sequences as predictors of later mathematical development. *Mathematics Education Research Journal, 33,* 513-540.

[17] Aubrey, C., Dahl, S., & Godfrey, R. (2006). Early mathematics development and later achievement: Further evidence. *Mathematics Education Research Journal, 18*(1), 27 – 46

Clements, D., Sarama, J., Spitler, M., Lange, A. & Wolfe, C. (2011). Mathematics learned by young children in an intervention based on learning trajectories: A large-scale cluster randomized trial. *Journal for Research in Mathematics Education, 42* (2), 127-166.

Mulligan, J. & Mitchelmore, M. (2009). Awareness of pattern and structure in early mathematical development. *Mathematics Education Research Journal, 21* (2), 33-49.

[18] Clements, D., Sarama, J., Baroody, A. & Joswick, C. (2020). Efficacy of a learning trajectory approach compared to teach-to-target approach for addition and subtraction. *ZDM, 52*, 637-648.

particularly in early mathematics learning[19], and a more specific curriculum can trace these connections, making them clear, and make sure their potential for improving learning is maximised.

2. Teacher expectations are critical to ākonga progress, and without clarity about what is possible, teachers' expectations can be too low.

There is evidence that ākonga are capable of more than most teachers expect of them. There is also evidence that teachers tend to expect less of Māori and Pacific ākonga than they are capable of, and tend to expect less of them than they might of ākonga of other ethnicities[20]. This is significant because of the impact of teacher expectation on learner progress and achievement, especially in mathematics[21]. By providing specification of what is possible to learn we can support teachers to have high expectations of all learners. To accelerate progress, the curriculum should make clear that more is possible in the first two years of schooling. Hiding these enhanced expectations in a three-year band risks continuing the underestimation of our ākonga, leading to insufficient progress and achievement – and ongoing inequity.

Support for teacher planning and teaching relates to the grain-size of curriculum detail. The current levels structure of the NZC has been problematic because of the large-grained curriculum information in the Achievement Objectives. The additional information provided by the Elaborations describes and explains detail of content, rather than expressing a learning pathway through the level. A year-by-year curriculum plan however provides finer grained detail so that teachers can more easily identify prior learning from previous years and know that ākonga have been taught this content. At present with the NZC levels, teachers cannot be sure what ākonga have been taught in the previous year, and so spend a great deal of time and effort in assessing and re-teaching. Another advantage of more fine-grained curriculum detail identifies the sequence of progress in each strand as well as between the strands[22]. Greater connections between the strands are important and can be made explicit, providing a platform for deeper conceptual understanding.

3. Teacher knowledge of mathematics and statistics for teaching is uneven, and because that knowledge is fundamental to improving learner outcomes, the curriculum needs to support and build teacher knowledge.

---

[19] Lowrie, T., Resnick, I., Harris, D., & Logan, T. (2020). In search of the mechanisms that enable transfer from spatial reasoning to mathematics understanding. *Mathematics Education Research Journal, 32,* 175-188.
Young-Loveridge, J. (2011). Rethinking the role of counting in mathematics learning. *Teachers and Curriculum, 12*, 79-83.
[20] Rubie-Davies, C. (2015). *Becoming a high-expectation teacher: Raising the bar.* Routledge.
[21] Rubie-Davies, C. (2015). *Becoming a high-expectation teacher: Raising the bar.* Routledge.
[22] Rowland, T. (2014). Mathematics teacher knowledge. In P. Andrews & T. Rowland (Eds.), *Masterclass in Mathematics Education: International perspectives on teaching and learning* (pp. 87-98). Bloomsbury Academic.

A more detailed and specific curriculum can support teachers' pedagogical content knowledge and knowledge of mathematics and statistics for teaching by organising and describing the key ideas in ways that support good design of learning experiences and formative assessment[23]. Quality opportunities to learn and feedback on learning are long-established tenets of equitable teacher practice, both of which require in-depth knowledge of content[24]. Providing a detailed framework for mathematics and statistics teaching can help teachers and leaders structure mathematics and statistics programmes in helpful ways. The clearer the curriculum is, the better targeted professional learning to improve teacher knowledge can be too.

4. The nature of mathematics and statistics as disciplines demands more detail than other curriculum areas

The structure of content (including process) for school mathematics and statistics is different to the structure of content (including process) for literacy, for example. Content of school mathematics and statistics has a structure that is both sequenced and interconnected. We draw an illustration from the number system (the parallel to the code of written language for literacy learning). Learning the code of how numbers are written is one part of the process, and the meaning of the number has to be understood in terms of its relative magnitude in the system of numbers. 28 represents a quantity but it is also in relation to one more than 27, and two less that 30 (where 30 is a benchmark number in the place value decimal structure of the number system). In literacy a word is a combination of letters that has a different relationship depending on context and meaning/s. For one more contrast with literacy, knowing to read from left to right and how to identify the letters of the alphabet (known as constrained skills) are claimed to be learned in a relatively short time. There is a much more generative aspect to literacy with more unconstrained learning needed for comprehension, speaking and writing. In contrast, when learning about number, ākonga learn about different types of numbers, and therefore different ways to write/encode numbers and number relationships. Knowing the place value system is not enough – there are decimal, fraction, negative numbers that make up a fundamental 'code' of numbers. The repertoire of constrained content in mathematics and statistics is substantial and typically needs to be sequenced across the years of the primary school.

We describe the mathematics and statistics experiences and understandings within *Te Whāriki* (Ministry of Education, 2017) that we can hope many students have on school entry, how we have considered the task of giving year-by-year indications of mathematics and statistics knowledge and understandings for Years 0-13, and key areas of focus leading these decisions. We used a guiding principle of each year level having some content that was identifiably new for ākonga (as well as material building on previous learning), and that teachers could extend learning

[23] Choppin, J. (2011). Learned adaptations: Teachers' understanding and use of curriculum resources. *Journal of Mathematics Teacher Education, 14*, 331-353.
Remillard, J. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75*(2), 211-246.
[24] Hattie, J. & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*(1), 81-112.

opportunities for ākonga within the year rather than moving into the next year level for extension ideas.

Many of the ‘what by when’ suggestions we offer in the year-by-year outlines below can be conceived of as whether or not ākonga are making links between powerful ideas, rather than whether they have or can perform from siloed items of knowledge. We need to check that ākonga can see these links, rather than viewing such signposts as simply not being able to perform specific skills. Key links, fundamental to sound number sense and good number understanding and arithmetic involve ‘composite units’ – things are made up of mathematical entities that can be split up and recombined according to rules – which can be numbers, shapes, patterns, seeing things and working on things in groups, and relative magnitude (e.g., whole numbers, rational numbers). The number line is a powerful model that can be used to understand composite units that also links to measurement scales and continuous models of number. Understanding that number is continuous, yet sometimes treated as discrete, and being able to see patterns and use them to understand and predict are all more significant for success in mathematics and statistics than just holding constituent knowledge or skills. Understanding of particular mathematics and statistics processes – such as ways of thinking, solving ‘problems’, communicating mathematically and statistically – and key competencies and cultural competencies are also important to embed throughout, so that the use and usefulness of mathematics and statistics are paramount, rather than separate, seemingly disparate, skills and strategies.

Students being able to see and make connections between mathematics and statistics ideas across the mathematics and statistics curriculum is associated with mathematical confidence, competence, understanding and engagement. Our suggestions of what mathematics and statistics is needed by when and our explanations of these demonstrate the importance of ākonga seeing and feeling confident about the interconnectedness of mathematics and statistics ideas across strands of the curriculum.

Reporting of ‘what by when’ - Explanation of style and format

Signposts

At each year level we have identified both the broad mathematical and statistical landscape for that year level and key learning that is required/needed/crucial for continued learning in the following or later years. We have named the key learning points as 'signposts' - places in the learning landscape that need particular attention and consolidation for future successful learning. The role of these signposts is to guide teachers when teaching this aspect of the curriculum - to concentrate attention but not take away learning time from other topics. As described above, the signposts give important direction, but their links to each other and to surrounding parts of mathematics and statistics are also critical for ākonga success.

A template used to organise presentation of these ideas was developed collaboratively by the report team. Each year level’s version of the template provides:

- overall emphasis for each strand each year
- a sketch of the mathematics and statistics landscape for that year, in the form of learning outcomes organised by strands (shown in normal text), preserving the sense of a rich learning experience
- 'what by when' aspects we feel would most hinder ākonga making further mathematics and statistics learning progress (signposts shown in bold)

Because we are concerned that the signposts could be seen as a checklist of skills we have placed them in their proper context in the template by:

- indicating the pervasive emphasis on mathematical processes, and
- showing a desire for a year-by-year thought/motto/whakataukī/kīwaha as a driving metaphor or statement of the year level's key mathematical and statistical contributions to the ākonga's learning journey.

Guiding focus for each year level

The 2007 curriculum was the first time that inclusion was made of a guiding whakataukī for our learning area, with one whakataukī to guide all levels. To build on this, a suitable next step would be to expand into adding a guiding whakataukī or motto, saying, kīwaha or the like for each year level. Each guiding whakataukī or motto, saying, kīwaha could be an inspirational focus for the mathematics and statistics teaching and learning at each year level that could be accompanied by a short paragraph highlighting key messages about mathematics and statistics teaching and learning of the year. These would ideally be determined as part of the curriculum writing process.

Preparing the templates

To start our work, each of the team completed the template for Year 7. Time has not been sufficient to develop shared views on all year-by-year content or to make all the year-by-year reporting similar in style. Rather, we offer the variety in style as each provides examples of what we have been asked to provide in different and useful ways. We completed templates for each year level, utlising the different knowledge and experience of the team. The templates are included below and overviews of continuity of overall ideas for each strand for each year level, taken from these, are included in Appendix One.

We have not carried out in depth consideration of the mathematical processes in the 'what by when' exercise or reporting, and hence these are left in each year-by-year overview as placeholders for the curriculum writers' further consideration. We believe there should be associated consideration of 'what-by-when' related to aspects outside of mathematics and statistics content, such as in progress with using mathematical processes, metacognitive processes, in mathematical and statistical self-efficacy, in literacy required for interpreting and solving problems, and in relation to engagement, for example. This work will be essential for ensuring ākonga success in the refreshed curriculum.

To give some indications of ways to consider the mathematical processes, examples are presented as three ‘stems’ where each combines important and related processes, to describe that we act and think mathematically and statistically in these ways, by:

- Making sense of and solving problems and using models, with perseverance
- Reasoning and arguing mathematically and statistically, with precision, and critiquing others’ reasoning
- Looking for, making use of, and communicating about patterns, regularity, and structure

These examples for describing mathematical processes encapsulate most of the key competencies in the 2007 curriculum, and in particular, the two competencies most closely connected with learning area content; ‘thinking’ and ‘using language, symbols, and texts’. The key competency of ‘thinking’ encompasses ways of thinking mathematically including being creative, being critical and being curious, contributing to success and enjoyment of learning. Not only are these important dispositions or orientations towards learning, for ākonga this can position them as active thinkers, of contributors to knowledge building in class settings, and for their engagement, belonging and connectedness. This focus also connects with current ideas of ‘future focused’, life-long learning, and being an engaged and critical citizen. The key competency of ‘using language, symbols, and texts’ is illustrated when ākonga at all years use both informal and formal language and symbolic forms in their mathematical activity. In a mathematics and statistics context, ‘encoding or decoding a text’ (using a literacy-based definition) relates to writing and reading texts in the form of sequences, equations, graphs (both algebraic and statistical), diagrams and other modelling representations. All of the above are encapsulated in the three mathematical processes and are also evident in the content included in the year-by-year templates below. The mathematical processes are pivotal to ākonga learning and signposts inclusive of processes and competencies are needed because the processes drive learning, engagement, and achievement. We have not had sufficient time to give full attention to signposts for the processes. However, some examples from a range of the year-by-year templates that illustrate how the mathematical processes are profoundly connected to important learning include:

In Year 3 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance: Use array models, number lines and materials that model the place value system to support problem solving and build understanding

Reason and argue mathematically and statistically, with precision, and critique others’ reasoning: Explain actions and ideas, ask another person for their ideas and be able to summarise what they said in reply

Look for, make use of, and communicate about patterns, regularity, and structure: Talk about and use array structures, the patterns in the place value system and patterns in basic facts, families of facts and multiples

Examples of signposts at Year 3 (from Year 3 template)

Number

Multiply single digit numbers and model the process using array models and other spatial models

Compare any two amounts and say which is bigger and by how much

Geometry
Make, copy and describe patterns that involve symmetry and rotation

Statistics
Asking questions and making a plan, gathering data, sorting data and explaining their reasoning

In Year 6 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance: Especially problems that embed multiplicative relationships and situations

Reason and argue mathematically and statistically, with precision, and critique others' reasoning: Explain, listen, summarise, question, build and critique ideas

Look for, make use of, and communicate about patterns, regularity, and structure: especially patterns in how the operations work and things can be seen as 'the same' in different ways, depending on what properties you are attending to

Examples of signposts at Year 6 (from Year 6 template)

Algebra
Use a table they are given to organise an explanation of a pattern that relates the term of the pattern to the pattern sequence (functional rather than recursive thinking)

Geometry
Use a co-ordinate system or the language of direction and distance to specify locations and describe paths

Statistics
Looking for patterns in the data, identifying the spread of data, and identifying and discussing patterns and trends in the data, (spread and outliers) and communicating our findings in text and visually

Rationale for content organisation: Number/Measurement, Algebra, Geometry and Statistics and Probability

Number and Algebra are individual strands so that each has a focus that can be differentiated and strengthened. Geometry is one strand - again to focus in on geometry content and the learning and teaching of geometry. Measurement has been moved to be considered within the Number strand. The rationale for this shift is that much of measurement learning relates to 'number in action', that is, the quantifying of measurements. This quantifying involves such actions as counting, repeating units in ones or tens, hundreds etc., (following the number system), reading scales (forms of number lines), and comparing measures (relative aspects of the attribute). Measurement provides both multiple contexts for using number (e.g., decimal numbers and parts of 'units' or wholes), and a system of measurement units based on the base ten number system (e.g., millimetres, centimetres, metres). This organisation brings discrete and continuous ways of quantifying into one strand, while highlighting powerful connections with other strands. For example, the measurement attributes of length, area and volume connect with spatial concepts/knowledge and visualisation. Counting and measurement data are part of the Statistics and Probability strand. Angle is part of the Geometry strand. Learning to 'tell the time' involves reading and interpreting numbers, and, for analogue clocks, connects with language and visualisation of simple fractions. The measurement processes of using, reading and making sense of scales, as well as knowing how to use measurement instruments are elements of mathematical practices of using tools and representations. Measurement processes also illustrate cultural experiences of engaging with and coming to know a sense of place – for example, a step or pace is known as hikoi, a concept also meaning shared walking together from one place to another for an important purpose. Similarly, aspects of 'measurement' sense such as using benchmarks (referents) to estimate measurements are mostly related to repeated whole or parts of the benchmark - whether by prior experience of the measurement of the benchmark (e.g., handspan) or using visual information that is at hand.

Rationale for inclusions and use of bold to show key signposts

We see what we are proposing as 'what by when' signposts in the templates as minimums rather than as targets. We want teachers to be encouraged and free to do more within the year, and not to do less than what has been suggested. We recognise that each part of the signposting has beginnings somewhere before the level it is included, which may or may not be explicit in the suggestions, and in most cases, later understanding will grow from aspects included earlier. One way we have framed the focus areas of most concern for further progressions is to show these in bold. A difficulty here is that a list of this sort might imply a narrowed curriculum, a check list of skills or high-stakes assessment foci, rather than being seen as signposts in a broad and rich mathematical and statistical landscape. Without a landscape to traverse, signposts are purposeless. In the same way, signposts important for progression are purposeless if they are not situated within a rich mathematical and statistical learning experience that creates interesting and valued pathways between the signs.

(i) Early Childhood Mathematics and Statistics – Prior to Year 0/1[25]

Years 0-13 mathematics builds on mathematics and statistics experiences and learning children have prior to entering school. Mathematics is an identifiable part of *Te Whāriki* the English version of our national early childhood curriculum. In contrast to the *The New Zealand Curriculum* (Ministry of Education, 2007) that groups understandings about the world in learning areas, in *Te Whāriki*, these understandings are woven through the strands of Mana Atua – wellbeing, Mana Whenua – belonging, Mana Tangata – contribution, Mana Reo - communication, and Mana Aotūroa – exploration. Mathematics is "explicit in communication and exploration" and "implicit in other strands" (p. 52). 'Young children' are the ages of approximately two and a half years to school entry, and their "developing literacy and mathematical abilities embrace new purposes, such as reasoning, verbal exploration, puzzling and find out about the physical and social world" (p. 15).

Examples of mathematically oriented Learning Outcomes in *Te Whāriki* include:

Recognising mathematical symbols and concepts and using them with enjoyment meaning and purpose | he kōrero pāngarau (p. 42)

Playing, imagining, inventing and experimenting | te whakaaro me te tūhurahura I te pūtaiao (p. 47).

[25] This material is based on the current English version of *Te Whāriki* (Ministry of Education, 2017) and other early childhood mathematical resources from the Ministry of Education, such as *Te Kākano* (Kei Tua o te Pae, Book 18, *Mathematics Pāngarau,* Ministry of Education, 2009) and *Te Aho Tukutuku* (2010).

Using a range of strategies for reasoning and problem solving | te Hīraurau hopanga (p. 47)

Examples of evidence of learning and development related to mathematics communication and exploration for Young children include:

Familiarity with numbers and their uses by exploring and observing the use of numbers in activities that have meaning and purpose (p. 42)

Ability to explore, enjoy and describe patterns and relationships related to quantity, number, measurement, shape and space (p. 42).

Recognition that numbers can amuse, delight, comfort, illuminate, inform and excite (p. 42)

which are evident when Young children:

have opportunities to learn numeric symbols and to use mathematical concepts and processes, such as volume, quantity, measurement, classifying, matching and pattern recognition (p. 44) … using such strategies as setting and solving problems, looking for patterns, classifying, guessing, using trial and error, observing, planning, comparing, explaining, engaging in reflective discussion and listening to stories (p. 47)

Young children are:

encouraged to use trial and error to find solutions to problems and to use previous experience as a basis for trying out alternative strategies. They are encouraged to give reasons for their choices and to argue logically (p. 49)

The mathematics framework of *Te Kākano* can be helpful to describe valued learning for four year old children. The strands of *Te Kākano* are described as purposeful activities, including playing – takaro, designing, tinkering, inventing – hanga, pattern "sniffing" – tauira, measuring – inenga, understanding symbols and representing – whakamarama tohu, noticing, recognising, and constructing relationships – nga panga/hononga, positioning/classifying/being systematic – whakatakotoranga/whakaropu, reasoning and comparing and using data – whakaaro whaitake, visualising and imagining – tukua nga whakaaro pangarua, locating – kimi/rapu, positioning/grouping/classifying – takotoranga/whaka ropu, estimating and predicting – matapae, and calculating and counting – tataihia kautehia. These activities are closely related to valued mathematical practices, set in meaningful contexts of children's interest and play, in either individual or shared activity. Understandably there is a range of prior experience and learning prior to school entry due to the diversity of family and community experiences as well as the wide range of early childhood settings.

Important mathematical learning for four-year-olds includes the following. Bold indicates the most important material important for progress in mathematics and statistics learning:

- Investigating patterns and collections of objects in purposeful/play contexts, leading to **forming patterns, extending, checking and repairing patterns**. These might be repeating patterns such as beading contexts, objects in repeated rows, music, dance, etc, recognising and repeating others' patterns.
- In collections of objects, **organising, categorising and ordering might lead to counting to find out 'how many' objects** in different groupings and understanding

more and less in relation to how many in sets of objects. A range of counting processes might be used including counting in ones, pairing and counting in twos etc.

- Related processes are comparing and noticing equal amounts/quantities, recognising numbers on dice and dominoes, leading to recognising visual patterns for numbers to 6 (early subitising). Estimating using a ‘thinking’ guess.
- Using their **home language for counting might occur during a counting process, within contexts of stories, rhymes and songs, and ritualised counting up to 5 (or more)**. Counting might be in a process of allocating objects for sharing or fairness. The number 5 is a significant benchmark number as four-year-olds approach their 5th birthday.
- Spatial patterns involve using two dimensional and three-dimensional space. Further spatial knowledge is evident in building/constructing models for purposeful play including fantasy play. **Using 3D materials such as blocks for constructions and using trial and error to position objects in space. Exploring with 2D shapes such as in paper folding and cutting, knowing names for simple shapes, moving, flipping and placing together shapes**, and using drawings to represent space. **These might be drawings of models, from stories, or maps of a built setting (the ece centre) or of a journey (based on a familiar walk or other journey)**.
- Comparing as a means of making sense of our world. Exploring quantities such as water, weight, height and other lengths, space etc. **Using language that describes sameness and difference and starting to see smaller lengths (or shapes) within larger lengths (or shapes).**

(ii) An overview of Year 0-6 mathematics and statistics decisions

Our intention is to signpost transition and continuity across sectors and this transition and continuity is in mind as informing our rationale for waiting until ākonga have been at primary school for two terms before identifying signposts. As an overall principle, our young ākonga are capable of more than we are asking them for at present. In determining a year-by-year approach for thinking about mathematics and statistics content, suitable key changes in pace and emphasis became apparent. These are outlined in detail in the year-by-year templates, and summarised here:

- We should ‘respond sooner’ by beginning after two terms at school rather than waiting for a year. This is intelligent noticing of what ākonga know so their progress can be enhanced if need be, rather than a labelling approach. It also enables two terms of transitioning time into a new school context, and can accommodate the different times of the school year that akonga begin primary school.
- The suggestions emphasise patterning and structure with a reduced emphasis on counting as the key route to arithmetic and number understanding. This is not a ‘pendulum swing’, it is acknowledging that counting is important and relevant, but research and our experience with counting-based number interventions, suggest it is not enough for providing secure foundations for mathematics and statistics learning. Ākonga need counting and pattern and structure.

- The idea of relative magnitude of numbers is key. Subitising and comparing numbers of things without counting are part of developing a sound understanding of relative magnitude.
- Ākonga who make the most progress in mathematics and statistics see links among the things they learn, recognising when something they know applies to a new situation - seeing how place value links to metric measurement for example - or recognising a 'rule' such as the commutative property and understanding it will always apply. We can help all ākonga by teaching such links as well as the skills and knowledge. Some of the signposts embed this idea – that ākonga need to learn relationships among the ideas, seeing patterns or applications across mathematical and statistical contexts. Ākonga need to able to use these features of the number system so that they can generate processes and solutions. They need to know how the number system 'works' – the structure of the number system, and the patterned way of thinking about place value and relationships between numbers, which is all part of having number sense. This idea could be further promoted in the way the curriculum itself is structured and expressed.
- The signposts suggest we start (and then expect) some things earlier than some current practice, including: grouping, sharing and the conceptual roots of multiplication, fractions, decimals and percentages, equality and functional thinking. They also provide guidance on when ākonga should have particular tools in their mathematics and statistics toolkit. In particular, ākonga should be using multiplicative reasoning across all their mathematics and statistics work by the end of Year 6. The evolution of this progress can be traced through the signposts which also suggest earlier competence with addition and subtraction, while making it clear that robust multiplicative reasoning does not grow out of additive reasoning, it needs to be encouraged in itself, hence the inclusion of signposts about arrays and fluency with multiples and factors.
- The indicators recognise the significance of spatial reasoning for progress and understanding in mathematics. Spatial reasoning seems to contribute to seeing pattern and structure in numbers and to developing ideas about relative magnitude using powerful spatial models such as number lines or place value blocks. There is evidence that spatial reasoning can be improved by teaching (i.e., it is not an innate, fixed ability). Therefore, attention is paid in the signposts to aspects of spatial reasoning and representations of number using that use space (such as arrays, number lines, growing patterns that use shape).

Our aim in the templates was to use language accessible to ākonga, whānau, and teachers.

I bring useful knowledge and understanding to my mathematics and statistics learning.

After two terms, the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

- Number and Measurement is refining and linking ideas of relative magnitude and subitising to the formal counting system
- Algebra is making patterns, and seeing patterns in things, including shapes and colours
- Geometry is making, seeing and talking about patterns in shapes
- Statistics and Probability is sorting things into categories that show something about the collection of things

After two terms at school we act and think mathematically and statistically in these ways:

- Make sense of and solve problems and use models, with perseverance
- Reason and argue mathematically and statistically, with precision, and critique others' reasoning
- Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

<table>
<tr><th></th><th>Number and Measurement</th><th>Algebra</th><th>Geometry</th><th>Statistics and Probability</th></tr>
<tr><td>Problems and models</td><td rowspan="3">⌛ Subitise to five using different ordered arrangements (tens frames, cards, dice)
⌛ Count and enumerate sets to at least 10
⌛ Compare two amounts (without counting) and say which is smaller or larger
⌛ Choose two cards that show 'the same' from a set with different pictorial representations of numbers (organised and not organised) and numerals, up to 5</td><td rowspan="3">♒ Continue a two-element repeating pattern</td><td rowspan="3">⌘ Copy a 2-D pattern with a line of symmetry, using a few simple shape blocks</td><td rowspan="3">⧫ Sort a pile of pattern blocks or cubes into colours and say something about what they've found</td></tr>
<tr><td>Reason and critique</td></tr>
<tr><td>Patterns and structure</td></tr>
</table>

In mathematics and statistics I am looking for patterns and the way things are related to each other.

In Year 1 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is understanding how amounts, numerals and number names are patterned and linked together, up to 100
Algebra is recognising what 'the same' means with amounts and shapes and looking for patterns in what they are learning
Geometry is being able to see and describe patterns that use simple symmetry and shapes
Statistics and Probability is using comparison to discuss investigations and their outcomes, including the language of chance

In Year 1 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

> At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique** **Patterns and structure**	⌛ **Start counting at any two digit number and count up from there** ⌛ Skip count in 2s, 5s and 10s ⌛ **Use subitising to identify groups within sets and find totals to 20** ⌛ Share amounts out fairly ⌛ Find ½ and ¼ of a group of objects or a region ⌛ **Join and separate groups and numbers** ⌛ **Match numerals with amounts and number names to 20** ⌛ **Order numerals and pictures of sets according to their relative magnitude (to 100 at least)** ⌛ **Order and compare objects by length, by direct comparison**	**Make repeating patterns and explain the pattern** Identify regularities and patterns within objects/pictures/arrays/patterns and in sequences of numbers **Identify and describe patterns in numbers to 100** **Identify things that are 'the same' (amounts, shapes in different orientations)**	⌘ **Create patterns using symmetry and explain their regularities** ⌘ Choose a shape from a group that matches a given description	⧫ **Discussing and telling and re-telling the stories of our investigations, using language of comparison** ⧫ Discussing and re-telling stories of their own explorations of statistical or probability contexts, and/or displays/drawings ⧫ Talking about things that might happen and possible outcomes ⧫ Asking questions and sorting into categories/groups ⧫ Counting and displaying data using people or objects, or drawings and pictographs

Whakataukī/Motto/Kīwaha/Saying **Year 2 Pāngarau Mathematics and Statistics Tauanga**

I can use pattern, number sense and simple statistical investigations to solve problems and learn new ideas.

In Year 2 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is developing a sound understanding of relative magnitude of numbers and begin to join, separate, compare and group to solve problems using this understanding as base

Algebra is seeking and describing patterns in the number system

Geometry is recognise, describe, and draw different shapes, and being able to see and talk about the effects of turns (rotation) and flips (reflections) on shapes

Statistics and Probability is working with others to pose a question, collect category data, sort it, discuss what was found and understand that not all responses or outcomes were equally likely

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

In Year 2 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance

Reason and argue mathematically and statistically, with precision, and critique others' reasoning

Look for, make use of, and communicate about patterns, regularity, and structure

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	⌛ **Start counting at any two digit number and count back from there** ⌛ Skip count forwards and backwards in 2, 5, 3 and 10 ⌛ **Add and subtract amounts - two digit numbers +/- single digit numbers** ⌛ **Know and describe the patterns in basic addition and subtraction facts** ⌛ Use patterns to extend addition and subtraction e.g., given 21+3 = 24, what is 31+3 etc ⌛ **Split and recombine numbers in groups e.g., split 20 into 10 lots of 2** ⌛ Find ½, ¼, ⅛, ⅓, ⅙ of a set or region ⌛ **Match numbers with representations of amounts and names to 100** ⌛ **Order amounts according to relative magnitude:**	**Identify and describe patterns in numbers to 1000** Create a growing pattern using materials	⌘ **Recreate a pattern they've been shown that uses symmetry and rotation** ⌘ **Solve spatial puzzles involving missing pieces and rotations** ⌘ **Rotate 2-D shapes mentally and physically, knowing what the resulting shape looks like** ⌘ Sort objects by their appearance. ⌘ Give and follow instructions for movement that involve distances, directions, and half or quarter turns. ⌘ Describe their position relative to a person or object. ⌘ Communicate and record the results of translations, reflections,	⧫ **Asking questions, collecting data and sorting into multiple categories** ⧫ Counting, comparing (what's the same and what's different), and discussing ⧫ Displaying data in simple tables and graphs e.g., pictographs, bar charts, simple stem and leaf ⧫ **Discussing and answering our initial questions, telling the stories of our investigations** ⧫ Listening to descriptions of others' investigations/simple displays. Counting, comparing (what's the same and what's different), and discussing

Patterns and structure	**whole numbers to 1000 and 1/2s and 1/4s** ⏳ **Put numbers on a number line to show relative magnitude** ⏳ **Compare two amounts to 20 and say which is bigger and by how much without counting** ⏳ **Order and compare objects by length, by counting whole numbers of units**		and rotations on plane shapes.	category data, using language of comparison. ⧫ Talk about what the outcomes of something might be and how predictable they might be

Whakataukī/Motto/Kīwaha/Saying **Year 3 Pāngarau Mathematics and Statistics Tauanga**

I understand how the place value structure of the number system works and can use this in exploring measurement and statistical situations.

In Year 3 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is understanding the place value system, and relate to number and measurement contexts
Algebra is knowing '=' means 'the same as' and being able to use it to show equalities
Geometry is moving, combining and drawing patterns of combinations of 2D shapes, and extend to moving 3-D objects in space
Statistics and Probability is working with more than one variable that describes a data set that they have collected and making sense of others' descriptions

In Year 3 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	**Have a useable and flexible understanding of the place value structure of whole numbers, explain the patterns and regularities that make it work** **Skip count forwards and backwards in multiples** **Recall basic addition and subtraction facts to solve problems** Use a number line to record skip counting, place numbers and fractions in order of relative magnitude, create and use a ruler **Group amounts using 10s and 1s and operate with 10s as a unit** **Know complementary numbers that add to 10** **Share and find the number of shares and how much is in each share** See, describe and use the structure of an array in rows and columns Find equivalent fractions for ½ **Multiply single digit numbers and model the process using array**	Make, extend and describe patterns that grow **Use = to show that things are the same**	**Make, copy and describe patterns that involve symmetry and rotation** **Solve 2-D and 3-D spatial puzzles with missing pieces** **Find or draw a shape that 'belongs' in a group of shapes** **Know position words and describe where things are in**	**Asking questions and making a plan, gathering data, sorting data and explaining their reasoning** Record results using tally marks or other. Organising and displaying data using tables or simple graphs such as pictographs, strip, bar charts Begin to collect measurement data Discussing our initial questions, revising our planning and communicating our findings **Interpreting visual information in others' statistical displays of data such as graphs**

Patterns and structure	models and other spatial models ⌛ **Compare any two amounts and say which is bigger and by how much** ⌛ **Describe the patterns in families of facts** ⌛ **Identify different characteristics of shapes, events and objects that can be measured** ⌛ Know ways to approach measuring length, area, volume and capacity, weight (mass), turn (angle), temperature, and time ⌛ Tell the time on analogue and digital clocks		**relation to each other**	⧫ Agreeing or disagreeing with statements made about data displays, and explaining with reasons. ⧫ Have vocabulary to discuss likelihood in everyday situations

I understand that there are patterns in operations on numbers and data displays.

In Year 4 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is developing consistently reliable methods for addition and subtraction, and beginning to understand rational numbers (including decimals and percentages) (how they show parts of a number or region or amount)

Algebra is developing a solid understanding of how addition and subtraction on whole numbers work - what they are asking you to do to numbers and what some of the underlying rules are about how they work

Geometry is knowing the spatial features that shapes and objects have and using these to sort and recognise shapes and objects in different places and orientations

Statistics and Probability is working with more than one variable that describes a data set collected by others and exploring the likelihood of particular outcomes

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

In Year 4 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance

Reason and argue mathematically and statistically, with precision, and critique others' reasoning

Look for, make use of, and communicate about patterns, regularity, and structure

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	⌛ **Have a reliable and accurate method for solving any addition and subtraction problem with whole numbers** ⌛ **Use principles and patterns to solve multiplication and division problems of different types with whole numbers, including measurement models of division** ⌛ **Put unit fractions on a number line** ⌛ **Understand non-unit fractions as parts of a region** ⌛ Express a proportion using a percentage ⌛ Use patterns and basic facts knowledge to find common multiples and factors of numbers ⌛ Use patterns and basic facts knowledge to find equivalent fractions	♒ **Use the inverse property to solve addition, subtraction, multiplication and division problems** ♒ **Use the commutative property to solve addition and multiplication problems** ♒ **Use 0 as the additive identity** ♒ **Use 1 as the multiplicative identity** ♒ **Use = to show that two**	⌘ **Sort objects by their spatial features, with justification** ⌘ Identify and describe the plane shapes found in objects ⌘ Create and use simple maps to show position and direction ⌘ Describe different views and pathways from locations on a map. ⌘ **Predict and communicate the results of translations, reflections, and**	⧫ Asking questions and making a plan, gathering and sorting data, making decisions about data. **Organising and displaying data using tables or graphs –** expanding range of possible displays, and explaining which display is best for their investigation, with reasoning (dot plots, stem and leaf, bar or pie charts, etc). Beginning to interpret the patterns in and shape of the data distribution ⧫ Discussing initial investigation questions, revising our planning and communicating our findings

	Use relationships between numbers to understand and solve problems e.g., the relationships among 2,3 and 6	**expressions are the same**	**rotations on plane shapes**	Interpreting visual information in others' statistical displays and critically consider statements about any patterns in the data
Patterns and structure	Work with tenths as an extension of place value knowledge Use appropriate devices to measure length, mass, area and volume			**Begin to identify advantages and disadvantages of simple graphs –** bar, pic and stem and leaf, with reasons Investigate simple situations that involve elements of chance, recognising equal and different likelihoods and acknowledging uncertainty

Whakataukī/Motto/Kīwaha/Saying **Year 5 Pāngarau Mathematics and Statistics Tauanga**

I am shifting to a multiplication-based understanding of how numbers work and use this to understand and express probability.

In Year 5 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

> Number and Measurement is establishing a method for multiplication and using place value, fraction and decimal understanding in measurement
> Algebra is using patterns to make predictions, and using inequalities to express relationships between numbers and expressions
> Geometry is describing and modelling equivalence and difference in the context of shapes as well as numbers
> Statistics and Probability is thinking critically about category data displays created by themselves and others

In Year 5 we act and think mathematically and statistically in these ways:

> Make sense of and solve problems and use models, with perseverance
> Reason and argue mathematically and statistically, with precision, and critique others' reasoning
> Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique** **Patterns and structure**	⌛ **Have a reliable and efficient, accurate method to solve multiplication problems, making sensible choices about methods (hand or machine)** ⌛ **Know basic multiplication facts and use them to solve more sophisticated problems** ⌛ Put unit and non-unit fractions on a number line ⌛ Put percentages on a number line ⌛ Work with decimals to 100ths as an extension of PV knowledge and percentages, and measurement ⌛ **Solve measurement problems using metric measurements for length and mass**	♒ **Continue a sequential number pattern and say why it continues that way** ♒ **Create and continue spatial patterns** ♒ **Use inequalities to express relationships between amounts and expressions**	⌘ **Use coordinates to define locations, and different ways of describing location and directions (compass points etc)** ⌘ **Be able to find invariant properties/equivalence between simple shapes and explain these using appropriate language** ⌘ **Identify which shape is a rotation or reflection of a specified shape from a range of choices**	⧫ Using existing data to expand range of display representations such as dot plots, back-to-back stem and leaf etc. Using digital technology such as spreadsheets to record data and generate data displays. **Identify possible advantages and disadvantages of different kinds of displays** ⧫ Using stem and leaf graphs to find the median and mode ⧫ Communicating interpretations of the data and posing more questions for investigation ⧫ **Investigating existing data – that are displayed in different ways – tables and statistical graphs** ⧫ **Asking relevant questions and making sense of the data** ⧫ Model the outcomes of a chance situation systematically

I am relating multiplication to place value, rational numbers, measurement systems, number patterns, geometrical patterns and relationships, data displays, probabilities, and different types of problem situations.

In Year 6 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is using multiplicative approaches to understanding place value, rational numbers and solving problems
Algebra is knowing how the four operations 'work' and beginning to use functional thinking to understand and describe two-variable patterns
Geometry is using formal language of geometrical properties to describe shapes, transformations and spatial positions
Statistics and Probability is collecting and using time series data as well as summary and comparison data, and being able to model the likelihood of outcomes in controlled probability situations (such as rolling two dice)

In Year 6 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

> At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	⌛ Solve problems involving rates, multiplicative comparison, part-whole multiplicative relationships, cartesian products and arrays, quotative and partitive division on whole numbers ⌛ **Approach multiplicative problems using multiplicative reasoning, not additive reasoning** ⌛ Understand improper fractions and put them on a number line ⌛ **Understand fractions as regional models, parts of amounts, products of division** ⌛ **Put any whole number, fraction, decimals or percentage on a number line to show relative magnitude** ⌛ **Understand the PV system for whole and**	≋ **Explain what's 'allowed' in each operation: commutative, distributive, associative and inverse properties** ≋ **Identify equality and equivalence in numbers, shapes and shape properties, fractions, rational number representations** ≋ **Use a table they are given to organise an explanation of a pattern that relates the term of the pattern to the pattern sequence**	⌘ Represent objects with drawings and models ⌘ **Classify plane shapes and prisms by their spatial features** ⌘ **Use a co-ordinate system or the language of direction and distance to specify locations and describe paths** ⌘ **Describe the transformations (reflection, rotation, translation, or enlargement) that**	⧫ Designing and carrying out all phases of a statistical investigation. Making decisions about data collection, and the organising and displaying data. Link reasons for decisions with parts of the statical investigation process such as the inquiry questions or the kinds of data. Choosing appropriate displays of data with reasoning, and include displays for multivariate and simple time series ⧫ **Looking for patterns in the data, identifying the spread of data, and identifying and discussing patterns and trends in the data, (spread and outliers) and communicating our findings in text and visually** ⧫ **Investigating existing data, including multivariate data – that are displayed**

	decimal numbers ⏳ Convert between metric measurement amounts, especially length: cm, mm, m, km	**(functional rather than recursive thinking)**	**have mapped one object onto another**	**in different ways – tables, graphically or other such as tree diagrams** ⧫ **Asking relevant questions, making sense of the data, and explaining choices of displays for different contexts** ⧫ Communicating interpretations of the data and posing more questions for investigation ⧫ Investigate simple situations that involve elements of chance by comparing experimental results with expectations from models of all the outcomes, acknowledging that samples vary
Patterns and structure	⏳ **Understand number as continuous** ⏳ Solve measurement problems using metric measurements for length, area, volume and capacity, weight (mass), ⏳ Find areas of rectangles and volumes of cuboids by applying multiplication. ⏳ **Develop benchmarks for conversion between fractions, decimals and percentages (equivalents for half, quarter, third, fifth, three quarters, tenth)**			

(iii) Years 7-10

The signposts for Years 7 and 8 have been aimed at encapsulating much of the current Level 4, but assuming that there has been stronger progress up to Year 6 than in current expectations. Years 9 and 10 content was informed using the current level 5 curriculum guidelines and relevant work in overseas curricula. Focus was given to providing examples of ways mathematical processes and digital technology and purposeful use of mathematics can be woven into statements of required content.

I can use my multiplicative reasoning, pattern recognition and number sense to solve problems, and to investigate statistical contexts and relationships, and critique solutions and conclusions.

In Year 7 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is solving problems using fractions, decimals, and percentages
Algebra is using tables, graphs and rules (words or numbers) to discover and describe sequential patterns
Geometry is using specific geometrical language that enables precision in describing and replicating shapes and design
Statistics and Probability is working with relationships in and through data, describing the shape of the data, and using continuous numerical variables

In Year 7 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique** **Patterns and structure**	⌛ **Recognise what operations to use in a problem involving whole numbers and use them accurately to find an answer. Use checking strategies** ⌛ **Use a range of estimation strategies for whole number situations** ⌛ Use fractions, decimals and percentages in contexts needing simple proportional reasoning ⌛ **Know equivalent forms of the same simple fractions** ⌛ **Know the equivalent representations and simple conversions for fractions, decimals and percentages** ⌛ Find percentages of amounts, including using calculator ⌛ **Use calculation strategies to find perimeters and areas of simple shapes – rectangles including squares** ⌛ **Read scales of measuring instruments using knowledge of place value and decimals with appropriate measurement units**	♒ **Recognise and using patterns in the number system to solve problems (odds and evens, multiplication patterns etc)** ♒ **Express number relationships with the equals sign representing equivalence on each side, for simple combinations of operations** ♒ **Represent number patterns in multiple ways – sequence, with words, models, tables, and simple graphs** ♒ Recognise and describe how to find a term in a pattern using words and numbers	⌘ Visualise, describe and make drawings and models of two and three dimensional shapes. ⌘ **Explore nets for cubes and cuboids** ⌘ Know and use geometrical properties to describe and classify two dimensional and three dimensional shapes, with reasoning ⌘ Know more complex shapes such as composite shapes and convex shapes ⌘ **Making sense of our environment using bearings, coordinates, distances and simple grid references** ⌘ Visualise, draw and describe transformations of reflection, translation, rotation and enlargement,	⧫ **Analyse the shape of the data distribution and identify the range, and measures of centre – median, mean and mode** ⧫ **Communicate interpretations of data and pose questions for investigation** ⧫ Investigate existing data, including multivariate data – displayed in different ways – e.g., using tables and graphs ⧫ Make sense of a greater range of presentations of statistical data from the local community or within the media (such as displays , interpretations and statements). Begin to ask

	⌛ Begin to consider precision of measurement scales		with increasingly complex designs and patterns	critical questions of the presentation of data and findings

I can reason proportionally using relationships and properties of number, shape and data.

In Year 8 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is choosing appropriate calculations and number representations to solve whole number, rational number and integer problems

Algebra is representing and explaining mathematical relationships in different ways, including using letters to represent variables and unknowns

Geometry is understanding features of shapes and space and moving between different models/representations of 2 and 3D shapes and objects

Statistics and Probability is engaging in all phases of a simple statistical investigation involving comparison or relationship situations, using and critiquing their own data sets and data sets from others, and be able to explain why the results of a probability experiment might not be the same as a model would predict

> At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

In Year 8 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance

Reason and argue mathematically and statistically, with precision, and critique others' reasoning

Look for, make use of, and communicate about patterns, regularity, and structure

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	⌛ Rational numbers can be represented and operated on in a variety of ways to solve problems. ⌛ Add/sub/mult/div on whole numbers, fractions, decimals, integers (may have to specify??) ⌛ **Choosing the best calculation process for different numbers and/or different contexts. Consolidating a range of checking processes and recognising when estimation should be used** ⌛ Competently use properties of the number system in different situations – e.g., distributive, associative, identity properties	♒ Be able to use symbols to express a function rule describing a pattern, and to graph this relationship ♒ **Using tables, graphs and equations to represent a linear number pattern. Explain the mathematical relationship and the advantages and features of each representation** ♒ Use letters to represent unknowns and variables in expressions (letter is the "number of' where students choose the letter with reasons) ♒ Using one of the above	⌘ **Consolidating knowledge of transformations – reflection with a range of mirror lines/axis of symmetry. Transformations of more complex shapes such as composite shapes** ⌘ Use proportional reasoning to understand geometrical relationships such as scaling (enlargement and reduction) ⌘ **Designing and testing ways of representing 3D shapes with 2D models (nets and plan views). Increasing confidence with the relationships between faces, edges and**	♦ **Plan all phases of a statistical investigation, making decisions about variables and data collection methods. Organising and displaying data using tables or graphs – multivariate and simple time series** ♦ **Using technology where appropriate for organising and displaying data** ♦ **Identifying patterns in the data including spread and patterns at the centre, and communicating our findings. Visually, verbally and with written explanations.** ♦ Expanding the range of ways that statistics is presented in everyday contexts. Looking for statistically sound

<table>
<tr>
<td></td>
<td rowspan="2">Converting between fraction, decimal and percentage representations in purposeful contexts.
Solve (simple) ratio problems using proportional reasoning
Put integers on a number line
Use appropriate scales, devices, and metric units for length, area, volume and capacity, weight (mass), temperature, angle, and time
Confidently convert between units showing a place value understanding for large numbers up to one million, and small numbers to thousandth. Relate to everyday uses of measurements and their units</td>
<td rowspan="2">representations to answer ‘what if’ questions
Writing mathematical equations in different ways showing an understanding of the equals sign and a balancing of each side (for equations that balance)</td>
<td rowspan="2">corners for 3D for a range of prisms
Knowing the 8 points of the compass and being able to read maps, interpreting direction and distance
Exploring relationships between area of a rectangle and the area of parallelogram, triangle, trapezium, and kite, by folding, or cutting. (not by formulas) explaining the relationships in words and visually</td>
<td rowspan="2">statements based on the data presented.
Also consider if there might be anything missing or alternative findings from the data.
Probability
Investigating a greater range of chance experiments such as two stage situations, with repeated trials, using technology or other methods to generate a very large number of trials. Exploring the concept of independent events in the context of their probability experiment. Making statements about their conclusions, with reasoning, including about the theoretical probability.</td>
</tr>
<tr>
<td>Patterns and structure</td>
</tr>
</table>

I can use strong content knowledge to select appropriate solution processes in a range of mathematics and statistics contexts.

In Year 9 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is developing confidence in their use of adding, subtracting, multiplying and dividing proportions, and in estimating and measuring
Algebra is making and solving algebraic equations and working with linear functions to solve practical problems
Geometry is using technology and geometrical relationships to resolve real world problems involving 2D shapes, scale and symmetry
Statistics and Probability is understanding that variation and chance affect results of statistical investigations and probabilities of events happening

In Year 9 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	⌛ Know fraction, decimal and percentage conversions, and flexibly apply to different contexts ⌛ **Confidently add and subtract fractions with like and unlike denominators** ⌛ **Find and use ratios to solve real life problems** ⌛ **Consolidate calculation processes with integers including many factor multiplication of negative integers** ⌛ **Expand use of rounding, significant figures, and standard form to more complex situations** ⌛ Exploring how whole numbers can be represented by prime numbers, by powers	**Be able to analyse linear patterns using tables, diagrams, and graphs, and find the general rule of linear patterns and equations of lines, and use the rule/equation to find unknown terms and make predictions of real life situations involving linear patterns** Be able to expand and factorise brackets in linear equations and identify and collect like terms and describe the mathematical properties that enable this	⌘ **Understand and use Pythagoras' rule to find lengths of side lengths of right angled triangles** ⌘ Use terms for and properties of angles on parallel lines and similar figures to draw and justify conclusions ⌘ Create accurate nets for simple polyhedra for purpose ⌘ **Compare properties of 3D shapes including in relation to their purposes of these shapes in everyday objects** ⌘ **Make and use scale drawings and**	Statistical Investigations ⧫ **Plan and carry out surveys using the statistical enquiry cycle to explore summary type questions** -determining appropriate variables and measures; -considering sources of variation; -gathering data; -using multiple displays, and re-categorising data to find patterns, variations, relationships, and trends in multivariate data sets;

Patterns and structure	of 2 etc. Generating different sequences of numbers ⌛ **Estimate, measure, calculate and use formulae for finding perimeters and areas of simple 2D shapes and combinations of these – both for abstract shapes and in real life contexts (parallelograms, quadrilaterals, triangles,)** ⌛ Exploring pi and using pi in circle problems. Find area and perimeter of composite shapes that involve circles or parts of circles. ⌛ Estimate, measure, calculate and use surface areas of 3D shapes and combinations of these for purpose (prisms, cuboids, sphere, cones, pyramids) ⌛ **Discuss and take into account accuracy of measurement devices and measurements made using these and consider implications of measurement errors in real life situations**	**Be able to confidently define terms and write equations that represent simple everyday linear situations, solve for unknown values, interpret the solution in light of the context, and justify and explain their mathematical thinking and decisions (e.g., perimeter and area of a new sports field)** Horizon knowledge: be able to show inequalities on number lines and graphs	⌘ **Represent 3d shapes in 2D using isometric paper and other methods** ⌘ Provide and use directions to follow a path using bearings and distances ⌘ Discuss properties of transformations, including congruence and invariance, and how these properties help us understand and use examples of transformations in everyday life ⌘ Discuss and compare properties of everyday objects in relation to how reflection, rotation, translation enable specific uses and artistic features	-comparing sample distributions visually, using measures of centre, spread, and proportion; -presenting a report of findings Probability ⧫ **Evaluate probability activities undertaken by others, including data collection methods, choice of measures, and validity of findings** ⧫ **Calculate probabilities, using fractions, percentages, and ratios**

I can use proportional reasoning in number, algebra, measurement, geometrical, and statistical contexts.

In Year 10 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Number and Measurement is being able to select from and use a range of number and measurement tools and strategies in number, measurement, algebraic, geometric and statistical contexts
Algebra is understanding relationships between information in tables, equations, and graphs for simple patterns and functions
Geometry is using technology, including geometry software, and geometrical relationships to understand and resolve real world problems involving 2D and 3D shapes, objects and symmetry
Statistics and Probability is understanding how the design of an experiment, data collection, sampling and analysis impact on findings of an investigation

In Year 10 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance
Reason and argue mathematically and statistically, with precision, and critique others' reasoning
Look for, make use of, and communicate about patterns, regularity, and structure

> At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique** **Patterns and structure**	⌛ **Investigating percentage change related to different contexts and critically discuss solutions, and implications for the context.** ⌛ Consolidate processes for finding and using rates and ratios. ⌛ **Explore multiplication of powers, and relate to exponential growth contexts. Also explore negative exponents.** ⌛ Move flexibly between squares and square roots, for a greater range of numbers, including decimals and fractions. ⌛ **Estimate, measure, calculate and use volumes of 3D shapes and combinations of these for purpose (prisms, cuboids, sphere, cones, pyramids)** ⌛ **Estimate, measure, calculate and use simple rates (e.g., distance covered in time taken, rate of flow or pour of**	♒ **Explore with and without software and discuss features of graphs of linear, quadratic, power and exponential functions to understand how gradient, intercepts, constants, turning points and sense are seen in graphs and function equations and to understand and make predictions about real life situations** ♒ **Be able to expand and factorise brackets in quadratic equations in order to solve and display quadratics**	⌘ Using trigonometric ratios with right angled triangles to find side lengths and angles ⌘ Deduce and use properties interior and exterior angles of polygons and use them to argue/justify responses ⌘ **Use coordinate planes, including paper based and software based coordinates, to investigate and describe properties of transformations, including congruence and invariance** ⌘ Connect three-dimensional solids with different two-dimensional representations ⌘ Construct and describe simple loci for functions, relations, and other geometrical and real life purposes	Statistics Investigations ♦ **Plan and carry out experiments using the statistical enquiry cycle to explore relational and comparative type questions** -determining appropriate variables and measures; -considering sources of variation; -gathering and cleaning data; -using multiple displays, and re-categorising data to find patterns, variations, relationships, and trends in multivariate data sets; -comparing sample distributions visually, using measures of centre, spread, and proportion; -presenting a report of findings Statistical Literacy ♦ **Evaluate statistical investigations undertaken by others, including data collection methods, choice of measures, and validity of findings**

	liquid, pace of speech in different languages) ⌛ Horizon knowledge: explore, describe, and consider reasons for the shapes of cross sections of three dimensional shapes	♒ Be able to change the subject of a formula ♒ Confidently add and subtract simple algebraic fractions	⌘ Use vectors to describe movements and combinations of movements (e.g, addition/subtraction of vectors and multiplication of vectors by numbers)	Probability ⧫ Compare and describe the variation between theoretical and experimental distributions in situations that involve elements of chance

(iv) Years 11-13

The signposts for year-by-year content for mathematics and statistics in years 11-13 are largely drawn from current practice. Deep consideration of suitable changes from this were not possible within the timeframe of this report preparation. Again, focus was given to providing examples of ways mathematical processes and digital technology and purposeful use of mathematics and statistics can be woven into statements of required content.

Recommendation:

- For Years 11-13, we strongly advocate for further urgent work on year-by-year signposting of mathematics and statistics content using a small team/s of experts working with an advisory/consulting group in an iterative consultative development process. Membership of these groups should include, for each of mathematics and statistics: at least one mathematics/statistics teacher, at least one mathematics/statistics university lecturer (with differing priorities for Year 11-13 curriculum content and emphasis), and at least one mathematics education researcher.

I can see and use links between different areas of mathematics and statistics.

In Year 11 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

> Number and Measurement is being able to confidently use all of the number and measurement tools and understandings needed for being a numerate citizen and for continued learning in mathematics and statistics
> Algebra is using equations, rules and graphs to explore real situations, solve problems and make predictions
> Geometry is selecting and using geometrical tools to investigate geometrical situations involving shape, position and symmetry across measurement, geometry and algebraic contexts
> Statistics and Probability is being aware of and able to critique chance and data situations relevant to important issues in their everyday lives and make decisions based on this awareness

In Year 11 we act and think mathematically and statistically in these ways:

> Make sense of and solve problems and use models, with perseverance
> Reason and argue mathematically and statistically, with precision, and critique others' reasoning
> Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

	Number and Measurement	Algebra	Geometry	Statistics and Probability
Problems and models **Reason and critique**	⌛ **Use multiple operations to interrogate and resolve real problems involving real numbers** ⌛ Solve context-based number problems using direct and inverse relations and powers with integers and fractions ⌛ Define and use irrational numbers (e.g., pi, square root of 2) ⌛ Investigate situations involving compound rates to draw conclusions ⌛ Use numerical strategies to find optimum values of functions ⌛ **Measure at a level of precision appropriate to the task and determine, explain and justify measurement accuracy (including accuracy of measuring devices, accuracy of derived measures, significant figures)** ⌛ Select and apply formulae relating to simple three-dimensional	♒ **Define variables, form and solve linear equations and inequations, quadratic and simple exponential equations, and simultaneous equations with two unknowns for purpose** ♒ Use digital tools to investigate, describe and visualise transformations to functions caused by adding or subtracting a constant to the function and within the function and by multiplying the function by a constant ♒ **With and without digital tools, use graphs, tables, and equations of linear, quadratic, and simple exponential relationships found in number and spatial patterns and in everyday contexts to explore situations and issues (including**	⌘ Deduce, apply and communicate using angle properties related to circles ⌘ Recognise when shapes are similar and use proportional reasoning to find unknown lengths in context-based problems ⌘ **Apply trigonometric ratios and Pythagoras' theorem in two and three dimensions in problem solving and modelling situations** ⌘ Use a co-ordinate plane or map to identify and use points in common and areas contained by two or more loci ⌘ **Visualise, compare and apply single and multiple transformations in art, mathematical and other real-world applications**	Statistical investigation ♦ Design and use a statistical investigation to integrate statistical and contextual knowledge to answer investigative questions important to people's lives and make informal inferences about populations from samples, justifying variables and measures used in the data collection phase and taking variation and uncertainty into account ♦ Use multiple displays to show and discuss features of the sample distributions in light of statistical and contextual information (e.g., trends, relationships between variables, differences within and between distributions) and potential implications of this information ♦ Use informal methods to compare populations using sample

Patterns and structure	figures (including prisms, pyramids, cones, spheres) to find unknown values ⌛ **Understand and use conversions between units to apply measurements to solve context-based problems**	**important societal issues) and make predictions** ♒ **Understand that the gradient of a graph gives the rate of change and use gradients to make predictions** ♒ **Use gradients and y-intercepts to create equations for real world situations** ♒ Generalise and be confident to apply the properties of operations with rational numbers, including the properties of exponents	⌘ Draw, describe and analyse symmetrical patterns using the transformations in them or used to create them	distributions (including reasoning about shift, overlap, sampling variability and sample size) Statistical literacy ⧫ **Interpret and evaluate informal everyday statistical reporting such as in advertisements and magazine articles in light of statistical processes and by relating the displays, statistics, processes, and probabilities used to the claims made** Probability ⧫ **Explore chance situations involving discrete random variables, understand the role sample size plays in estimating probabilities via experiment, record results and plot frequencies of outcomes and investigate probability distributions** ⧫ **Calculate probabilities in discrete situations and explain and justify whether theoretical chance outcomes are or are not equally likely**

I can use tools of mathematics and statistics to understanding and explore mathematical and real world problems.

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

In Year 12 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Mathematics is using multiple representations for an expanded range of mathematical expressions, and bringing together algebraic and geometric techniques to solve problems

Statistics and Probability is organising and displaying complex data sets and using statistical tools to critically interpret and report findings

In Year 12 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance

Reason and argue mathematically and statistically, with precision, and critique others' reasoning

Look for, make use of, and communicate about patterns, regularity, and structure

<table>
<tr><th></th><th>Mathematics</th><th>Statistics and Probability</th></tr>
<tr><td>Problems and models</td><td rowspan="3">Patterns and relationships
≈ Identify, describe and use arithmetic and geometric sequences and series to investigate mathematical and everyday patterns and make predictions with consideration of effects of real world variables
≈ Use digital tools to display the graphs of linear and non-linear functions and identify and discuss relationships between the structure of the functions with their graphs (including exponential, logarithmic and trigonometric equations (range 0-2π))
≈ Derive and use trigonometric relationships, including the sine and cosine rules, in two and three dimensions and right and non-right angles triangle, to calculate lengths and angles to help solve geometric problems, with justification and consideration of accuracy
≈ Confidently use and manipulate exponents, surds and logarithms as functions of numbers and of unknowns
≈ Design, choose and justify appropriate networks to find optimal solutions for practical situations
≈ Derive and use co-ordinate geometry techniques to find aspects of points and lines with justification

Equations and expressions
≈ Confidently manipulate rational, exponential, and logarithmic algebraic expressions
≈ Form and use linear, quadratic, and simple trigonometric equations with and without digital technology and with and without focus on real world applications, interpreting and verifying solutions
≈ Form and use digital technology to explore solutions of pairs of simultaneous equations, one of which may be non-linear for purpose

Calculus</td><td rowspan="3">Statistics investigations
♦ Use statistical investigations (with experimental and existing data sets) to explore and report on an important societal issue, make informal estimated comparison intervals for population parameters, informal predictions, interpolations, and extrapolations, considering sampling variability and sample size effects, and discuss implications of findings and potential recommendations for action
♦ Design a questionnaire specific to a given purpose and use random sampling techniques for data collection
♦ Use results from statistical investigations to explore and ascertain risk and relative risk and interpret and communicate about risk

Statistical literacy
♦ Interpret and evaluate formal statistical reporting such as in social polls and surveys in light of aspects of the statistical cycle and considering sampling and non-sampling errors and use interpretation and evaluation to make predictions and discuss implications and recommendations for action

Probability
♦ Investigate chance situations involving continuous variables comparing theoretical distributions (e.g., normal distribution) with experimental distributions</td></tr>
<tr><td>Reason and critique</td></tr>
<tr><td>Patterns and structure</td></tr>
</table>

	♒ **Investigate and describe the relationships between the graphs of functions and between the graphs of functions and their gradient functions (including through use of first principles), with and without digital technology and with and without focus on real world applications** ♒ Apply differentiation and anti-differentiation techniques to polynomials	♦ Use tools such as probability tree diagrams, two way tables, simulations and technology to calculate probabilities ♦ **Use technology-based and physically-based experiments to estimate probabilities**

Whakataukī/Motto/Kīwaha/Saying **Year 13 Pāngarau Mathematics and Statistics Tauanga**

I can use a range of sophisticated mathematics and statistics strategies and tools for exploring important mathematical and real world problems.

In Year 13 the main kaupapa/purpose/focus for ākonga of the teaching and learning of:

Mathematics is using digital and other tools to represent, compare and contrast, and generate an expanded range of mathematical relationships

Statistics and Probability is using experimental design principles and statistical tools and techniques to critically interpret, evaluate and report for complex data sets and for probability context

In Year 13 we act and think mathematically and statistically in these ways:

Make sense of and solve problems and use models, with perseverance

Reason and argue mathematically and statistically, with precision, and critique others' reasoning

Look for, make use of, and communicate about patterns, regularity, and structure

At all year levels, we learn mathematics and statistics to help understand and explain the world around us, make predictions about what might happen, influence our own and others' decision making, and positively impact on the future.

<table>
<tr><th></th><th>Mathematics</th><th>Statistics and Probability</th></tr>
<tr><td>Problems and models</td><td rowspan="3">Patterns and relationships
♒ Apply the geometry of conic sections in mathematical and real world contexts
♒ Use digital technology to investigate, display and interpret graphs of functions with graphs of their inverse and/or reciprocal functions
♒ Use permutations and combinations to find the count and for solving probability problems
♒ Use digital technology to use curve fitting, log modelling, and linear programming techniques for purpose
♒ Develop, use and justify the design of network diagrams to find optimal solutions, including critical paths

Equations and expressions
♒ Confidently manipulate and use trigonometric expressions
♒ Form and use trigonometric, polynomial, and other non-linear equations with and without digital technology and with and without focus on real world applications, interpreting and verifying solutions
♒ Form and use digital technology to explore systems of simultaneous equations, including three linear equations and three variables, and interpret the solutions in context
♒ Confidently manipulate complex numbers and present them graphically

Calculus
♒ Understand, describe and use connections between functions and gradient functions
♒ Identify and interpret discontinuities and limits of functions</td><td rowspan="3">Statistical investigations
♦ Use different data collection contexts using experimental design principles to integrate statistical and contextual knowledge to answer important investigative questions, use statistical models (e.g., including linear regression for bivariate data and additive models for time-series data), seeking explanations, and making predictions, make statistical inferences about populations or processes from samples (e.g., using bootstrapping or randomisation) to determine estimates, confidence intervals, forecasts, and strength of evidence, evaluating all stages of the cycle, and using this to discuss implications and recommendations
♦ Explain sampling variability and sample size effects

Statistical literacy
♦ Make inferences from surveys and experiments on important social contexts determining estimates and confidence intervals for means, proportions, and differences, recognising the relevance of the central limit theorem and using methods such as resampling or randomisation to assess the strength of evidence
♦ Evaluate a wide range of statistically based reports, including surveys and polls, experiments, and observational studies and critique causal-relationships claims and interpret margins of error

Probability
♦ Investigate chance situations using probability concepts and distributions including using concepts such as randomness, probabilities of combined events and mutually exclusive events, independence, conditional</td></tr>
<tr><td>Reason and critique</td></tr>
<tr><td>Patterns and structure</td></tr>
</table>

	♒ **Choose and apply a variety of differentiation, integration, and anti-differentiation techniques to functions and relations, using both analytical and numerical methods for mathematical and real world problems, with and without digital technology, interpreting and verifying solutions** ♒ **Form and justify differential equations for modelling simple mathematical and real world situations and interpret the solutions**	probabilities and expected values and standard deviations of discrete random variables, and probability distributions including the Poisson, binomial and normal distributions ♦ **Model chance situations using discrete and continuous probability distributions** ♦ **Use probabilities from theoretical models and estimate probabilities using experiments**

Pedagogy - Important cross-discipline links

We have considered important cross-disciplinary links between mathematics and statistics and other disciplines in three ways:

- how suitable material from other disciplines can be reinforced within a mathematics teaching and learning programme (e.g., see initial approach for considering this in relation to te reo Māori, Appendix Two)
- how suitable material from mathematics and statistics can be reinforced within teaching and learning programmes of other disciplines
- how mathematics and statistics and other disciplines can be taught in integrated topics, (i.e., topics which are designed and taught to encompass teaching and learning from two or more disciplines).

(i) Supporting teaching of material from other disciplines within mathematics and statistics teaching and learning programmes

The main focus of mathematics and statistics teaching is developing mathematics and statistics learning. At times, there are good opportunities to support the development of understanding of material from other disciplines within this. Links with the teaching and learning of other disciplines are particularly useful when they help develop mathematical and statistical knowledge and understanding, awareness of the real world uses of mathematics and statistics and engagement with and motivation for learning mathematics and statistics. Making such links effectively and with integrity in teaching and learning programmes requires sound content and pedagogical content knowledge both of mathematics and statistics and the discipline being linked with.

Important opportunities for supporting learning of other disciplines within mathematics and statistics teaching and learning include:

- teaching and using te reo Māori related to the mathematics and/or statistics learning purpose (e.g., see Appendix Two)
- teaching using science investigations as contexts for reinforcing understanding of measurement and emphasising key ideas, such as accuracy, estimation, reading scales, use of rational numbers, use and interpret data display for category or measurement data, understanding that a sample may or may not represent the actual population characteristics and the usefulness of repeating procedures for increasing certainty
- teaching using a technology curriculum emphasis e.g., using measurement to help solve a design-based problems to meet specifications, using knowledge of position and transformation to programme simple paths
- teaching using a literacy and English curriculum emphasis, such as through using critical evaluation of claims made in persuasive texts and media using understanding of statistics and percentages

- reinforcing number and use and interpretation of data and probabilities within health-related contexts and the social sciences to critique claims and data displays, reflect on whose interests are served in how the numbers are presented, make judgements and explore and understand consequences
- reinforcing number and measurement ideas through examples from physical education, such as being able to accurately measure, understand and use measurement data (e.g., heart rate change)
- reinforcing shape and position concepts in creating, understanding, describing and interpreting visual art, dance and music
- reinforcing use and interpretation of appropriate data display for category or measurement data within social studies, geography and history
- reinforcing number and data to critique claims and data displays, thinking about whose interests are served in how the numbers are presented
- link fraction knowledge to musical notation
- reinforcing transformation geometry ideas within teaching of art

(ii) Supporting teaching of material from mathematics and statistics within teaching and learning programmes of other disciplines

Aspects of mathematics and statistics are often used in the teaching and learning of other disciplines. However, the main focus of such teaching is the discipline other than mathematics and statistics, so the learning intentions, learning experiences and assessment are focussed on content of the other discipline. In our experience, opportunities for supporting learning of mathematics and statistics within other disciplines most often involve the application of selected mathematical and statistical knowledge, tools, and strategies from Number, Measurement and Statistics, as necessary for the learning material of the other discipline. Examples from Number include use of large and small real, fractional, and decimal numbers, proportions, percentages, operations on these, and rates. Examples from Measurement include making and reporting measurements, using scales, considering accuracy and making estimates of measures. Examples from statistics include sorting, analysing, and displaying data, although often with graphs and values that are different from those in the current statistics curriculum. In such teaching, teachers largely make use of the understanding students bring, rather than teach these areas of mathematics and statistics within their teaching of other discipline/s. In such instances, transfer of learning from one context to another is often a problem, in that ākonga can do something ‘in maths time’ that they can’t or won’t draw on in ‘science time’. This issue can be deliberately addressed in both contexts by teaching for using these links, including through context-based teaching, while keeping the integrity of the mathematics and statistics.

While mathematics and statistics knowledge and skills are likely to be supported when they are used in the service of other disciplines, the understanding necessary for deep learning, retention and understanding of the important connected nature across mathematical and statistical ideas are unlikely to be able to be developed well for students when the focus of the work is on another discipline. Again, making such links with effectiveness and integrity in teaching and learning programmes requires sound content and pedagogical content knowledge of the discipline being taught and mathematics and statistics.

(iii) Teaching mathematics and statistics within Integrated learning programmes

There is already much written about integrated learning and the affordances and challenges of this teaching approach.

Whichever approach is used (i-iii), maintenance of the integrity of each discipline in terms of curriculum emphasis, content and pedagogy is essential. For example, primary teachers are well versed in the content and emphases of the various curriculum areas and are well placed to make such links in their teaching in ways that support the learning of the other disciplines, and that support the learners they are working with. In our experience, many secondary teachers are less aware of curriculum-specific emphases outside of their specialism/s. There is a danger that cross-discipline links may do more harm than good if they lead to confusion and undermining of students' understanding and confidence in what they have learnt.

Several further considerations are also important when considering cross-discipline links in relation to mathematics and statistics. Most teachers were taught mathematics and statistics prior to the current mathematics and statistics emphases, and while primary teachers have had professional development and resources to assist them with these, secondary teachers of disciplines other than mathematics and statistics will not necessarily be familiar with current thinking about mathematics and statistics content and teaching and current digital technologies important for learning and doing mathematics and statistics. Secondly, consideration must be given to the cognitive load for students which is likely to be higher while trying to develop mathematics or statistics understanding at the same time as new learning within another discipline. Teaching from more than one discipline at a time is also more challenging for teachers, given the deeper calls on pedagogical content knowledge required for teaching from more than one discipline. In compiling this report, our reflection is that greater 'within' subject integration is possible and necessary for enhancing mathematics and statistics teaching and learning. 'Intra' subject links, rather than 'inter' subject links are even more vital now that research and curriculum are recognising greater links between number, statistical thinking, patterning, and spatial thinking.

Recommendations:
In summary, our response regarding important cross-disciplinary links between mathematics and statistics and other disciplines is that such links should only be used by teachers when the integrity of the curriculum emphasis and content of each discipline is maintained. For mathematics and statistics, this includes emphasis on important links within the discipline. Cross-discipline links could also be used to provide pleasure in learning, engagement, or motivation. However, inter-disciplinary links should be approached with caution, including with consideration for managing students' cognitive load and provision of additional planning time for teachers.

Pedagogy - Considering technology throughout all year levels and strands and in relation to rapid changes and growth in computer science/ICT

This report takes a broad view of technology to include invented tools that can be used to support the learning and doing of mathematics and statistics. With this definition of technology as invented tools, culturally-linked important ways of describing, measuring and representing our world(s), are examples of technologies invented and tested over time that can be explored and used (e.g., ways of finding locations and describing 'place').

Technology is broader than digital technology, including physical materials, structured representations, and equipment. Physical materials such as blocks and tiles can serve multiple learning purposes – as models representing contexts or as elements for constructions. Technology includes measuring instruments including rulers, weighing scales and clocks (analogue and digital). Incorporating these technologies into mathematical learning leads to knowing how to use and 'read' the scales of these instruments and generating measurements that contribute to further mathematical activity/learning in various areas. Similarly, grid patterns, arrays and even a construction such as a 'One Hundred Board' is a technology - a stable representation that helps organise and present important mathematical entities for learners, such as the place value system. In each case, introduction and use of the tool is needed to enable effective use of the tool to explore and interrogate mathematical or statistics ideas.

The role of technology is to help learners 'get into' important mathematics and statistics and to create pathways that can help bypass tedious calculations that can disrupt more important thinking. The role of technology is not to mask or trivialise the powerful mathematics and statistics being used or to 'get out of doing' mathematics and statistics. The important role of technology is in re-presenting information, often visual, in a new way, or organising and displaying information for further learning (examples are patterns and structure of mathematical entities, and statistical displays etc.). For example, simple calculators can act as a 'cognitive friend' for younger children by being a number generation machine that offloads some of the 'memory' effort and cognitive load. At higher levels of the school, technology such as spreadsheets and graphing technologies are digital tools for organising and manipulating large data sets, generating more complex models and providing a range of interfaces for students to engage with. Suitable technological tools are an important part of mathematics and statistics learning throughout Years 0-13. This view is implied in the Year level templates. As we are aware there is some variation in practice and philosophy, in some of the templates for higher year levels, we have explicitly indicated examples of where we feel some content should be explored with digital technology or with and without digital technology.

Recommendation:

- We strongly advocate for ready and equitable access to and use of powerful technologies throughout Years 0-13 suitable for exploring and using the mathematics and statistics included at each level.
- That the refreshed curriculum be explicit about the benefits of young children engaging with physical models of mathematical and statistical ideas as well as using digital tools.

Computational thinking - Aspects of computational thinking are present, but in the background, in the current learning area of mathematics and statistics. Examples of activities where this is evident but not identified as 'computational thinking' include; The Sieve of Erastothenes (Number strand), setting up a spreadsheet (Algebra strand for patterns, tables, and graphs), instructions for a journey/network map (location - Geometry), metaphor of a number machine (Number and Algebra), steps for combining transformations to generate complex shapes for further transformation or tessellation (Geometry), and in the senior secondary school, spreadsheets, graphing software/graphical calculator provide statistical modelling tools to use with complex data sets. An approach that brings forward aspects of computational thinking, without overlap with the *Digital Technologies curriculum*, will better focus on the process aspects related to modelling and solving complex problems. Flow charts are accessible visual tools for supporting this process. Computational thinking involves processes of decomposition - breaking a complex problem down into parts, finding a pattern that shows a logical sequence of steps to be followed and possibly repeated, and testing and refining the process (sometimes producing an algorithm). This approach to modelling situations within existing mathematical and statistical content is a possible way forward for including a focus on computational thinking within the curriculum refresh.

Pedagogy - Further thoughts on pedagogy suitable for teaching and learning mathematics and statistics

This report has been prepared at time when there have been strong but mixed messages regarding suitable teaching methods for mathematics and statistics and therefore some confusion amongst teachers, families and the wider public. In addition, since the 2007 curriculum further tools and expectations of teachers regarding pedagogy and interactions with ākonga and families, such as *Tātaiako* and *Tapasā*, have been provided to teachers, and further literature on culturally sustaining, responsive and relevant pedagogy and assessment is now available. Ākonga experiences of learning the 'what by when' of the curriculum are heavily influenced by the pedagogies kaiako use. While consideration of pedagogy is largely outside the brief for this report, thinking about suitable pedagogy for enhancing access to learning and reducing barriers to learning has influenced our thinking and responses. Hence, we have included some emphasis consistent with suitable and equitable pedagogies throughout this report. To be clear, there is no one way to effectively teach mathematics and statistic. Students are best served when they are able to explore mathematical and statistical ideas in many diverse individual, pair-based and collective ways. They are underserved in learning situations in which only a limited range of pedagogies are present.

Recommendations:

- We strongly advocate for a clear national statement about the use of pedagogies suitable for teaching ākonga mathematics and statistics, consistent with *Tātaiako* and *Tapasā* and current thinking about culturally sustaining pedagogy.
- We strongly advocate for further information and guidance for teachers regarding effective pedagogies for teaching and learning of mathematics and statistics to be included in the curriculum writers' brief and work.

Final comments

This report addresses the Ministry of Education's request for advice on the **mathematics skills and knowledge learners need to know and by when, important cross-disciplinary links, and considerations in light of rapid changes and growth in computer science/ICT.** In doing so it also goes beyond this brief by surrounding this information with a number of key considerations and additional information. This is because the task of producing a 'what by when' list is somewhat fraught: such a list can easily become a checklist of skills taught in isolation, the foundation of high stakes testing or reporting or a proxy for the curriculum. In practice the key skills and concepts will not emerge unless they are taught in a rich and broad mathematics and statistics learning experience because many of them are about joining ideas together, recognising patterns, similarities, and the power of mathematical structures. For these reasons, users of this document should guard against extracting a shortlist of our signpost suggestions. We ask readers to consider this work in relation to the metaphor of a landscape introduced earlier with our suggestions as signposts.

We repeat here the caveats on this report arising from the way it was derived. In particular, this document does not represent a bicultural view of these issues. Notions of progress, development and cross-disciplinarity have cultural underpinnings and these have not been discussed or challenged from a Māori perspective, nor is the template framework used bicultural at its foundations. The Ministry of Education needs to remember this when it uses the information in this report. We do not want our efforts to undermine a genuinely bicultural development process for the curriculum, which we completely support. Our report draws from and is consistent with themes, findings, and key ideas from mathematics and statistics education research and literature. However, the time given for preparation of this report has not enabled us to do justice to suitably showing how selections of literature have informed this writing. In addition, consultation on our suggestions with an appropriate reference group has not been possible. We have not been able to consult with a range of others to get diverse views on our suggestions. Appendices Three and Four contain commentary on statistics and senior mathematics from some key thinkers in these areas. These reflections contain useful information for the next step in the curriculum writing process and are therefore shared in full in the appendices.

The Ministry of Education has the difficult task of drawing together a range of advice from different sources to refresh the mathematics and statistics curriculum. We have tried to dovetail our suggestions with some of the sources we had access to. Coherence across advice to teachers and between curriculum and assessment is going to be critical to ākonga success as a result of the curriculum refresh. One difficulty posed by the mathematics and statistics curriculum area is that there is clear evidence that two-year long levels is contributing to slow progress for our ākonga. Moving to longer bands of time will exacerbate this. A means needs to be found for mathematics and statistics to specify goals and minimum outcomes for each year of schooling (and after two terms at school) within the need for consistency of presentation across curriculum areas.

Finally, this report represents our best thinking within the timeframe allowed. It should be seen as a starting point for further work and discussion rather than a complete and flawless 'master list' of ideas.

References

Education Council New Zealand / Matatū Aotearoa. (2011). *Tātaiako: Cultural competencies for teachers of Māori learners.* Wellington, New Zealand: Ministry of Education.

Education Council New Zealand / Matatū Aotearoa. (2017). *Our code, our standards.* Wellington: Ministry of Education.

Ministry of Education. (2007). *The New Zealand curriculum for English-medium teaching and learning in years 1-13.* Wellington: Learning Media Ltd.

Ministry of Education. (2017). *Te whāriki.* Wellington: Ministry of Education.

Ministry of Education. (2018). *Tapasā: Cultural competencies framework for teachers of Pacific learners.* Wellington: Ministry of Education.

Appendix One: Continuity Overview Tables

These tables provide information extracted from the templates in a year-by-year format to make the focus suggestions easier to see as a progression. The statements are too broad to be used as signposts because each of them implies a number of ideas and skills that ākonga need. These are umbrella expressions of progression to give context to the signposts.

	Year's ākonga motto
Two terms	**I bring useful knowledge and understanding to my mathematics and statistics learning.**
Year 1	**In mathematics and statistics I am looking for patterns and the way things are related to each other.**
Year 2	**I can use pattern, number sense and simple statistical investigations to solve problems and learn new ideas.**
Year 3	**I understand how the place value structure of the number system works and can use this in exploring measurement and statistical situations.**
Year 4	**I understand that there are patterns in operations on numbers and data displays.**
Year 5	**I am shifting to a multiplication-based understanding of how numbers work and use this to understand and express probability.**
Year 6	**I am relating multiplication to place value, rational numbers, measurement systems, number patterns, geometrical patterns and relationships, data displays, probabilities, and different types of problem situations.**
Year 7	**I can use my multiplicative reasoning, pattern recognition and number sense to solve problems, and to investigate statistical contexts and relationships, and critique solutions and conclusions.**
Year 8	**I can reason proportionally using relationships and properties of number, shape and data.**
Year 9	**I can use strong content knowledge to select appropriate solution processes in a range of mathematics and statistics contexts.**
Year 10	**I can use proportional reasoning in number, algebra, measurement, geometrical, and statistical contexts.**
Year 11	**I can see and use links between different areas of mathematics and statistics.**
Year 12	**I can use tools of mathematics and statistics to understanding and explore mathematical and real world problems.**
Year 13	**I can use a range of sophisticated mathematics and statistics strategies and tools for exploring important mathematical and real world problems.**

Number and measurement

	Main kaupapa/purpose/focus for ākonga
Two terms	refining and linking ideas of relative magnitude and subitising to the formal counting system
Year 1	understanding how amounts, numerals and number names are patterned and linked together, up to 100
Year 2	developing a sound understanding of relative magnitude of numbers and begin to join, separate, compare and group to solve problems using this understanding as base
Year 3	understanding the place value system, and relate to number and measurement contexts
Year 4	developing consistently reliable methods for addition and subtraction, and beginning to understand rational numbers (including decimals and percentages) (how they show parts of a number or region or amount)
Year 5	establishing a method for multiplication and using place value, fraction and decimal understanding in measurement
Year 6	using multiplicative approaches to understanding place value, rational numbers and solving problems
Year 7	solving problems using fractions, decimals, and percentages
Year 8	choosing appropriate calculations and number representations to solve whole number, rational number and integer problems
Year 9	developing confidence in their use of adding, subtracting, multiplying and dividing proportions, and in estimating and measuring
Year 10	being able to select from and use a range of number and measurement tools and strategies in number, measurement, algebraic, geometric and statistical contexts
Year 11	being able to confidently use all of the number tools and understandings needed for being a numerate citizen and for continued learning in mathematics and statistics

Algebra

	Main kaupapa/purpose/focus for ākonga
Two terms	making patterns, and seeing patterns in things, including shapes and colours
Year 1	recognising what ‘the same’ means with amounts and shapes and looking for patterns in what they are learning
Year 2	seeking and describing patterns in the number system
Year 3	knowing ‘=’ means ‘the same as’ and being able to use it to show equalities
Year 4	developing a solid understanding of how addition and subtraction on whole numbers work - what they are asking you to do to numbers and what some of the underlying rules are about how they work
Year 5	using patterns to make predictions, and using inequalities to express relationships between numbers and expressions
Year 6	knowing how the four operations ‘work’ and beginning to use functional thinking to understand and describe two-variable patterns
Year 7	using tables, graphs and rules (words or numbers) to discover and describe sequential patterns
Year 8	representing and explaining mathematical relationships in different ways, including using letters to represent variables and unknowns
Year 9	making and solving algebraic equations and working with linear functions to solve practical problems
Year 10	understanding relationships between information in tables, equations, and graphs for simple patterns and functions
Year 11	using equations, rules and graphs to explore real situations, solve problems and make predictions

Geometry

	Main kaupapa/purpose/focus for ākonga
Two terms	making, seeing and talking about patterns in shapes
Year 1	being able to see and describe patterns that use simple symmetry and shapes
Year 2	recognise, describe, and draw different shapes, and being able to see and talk about the effects of turns (rotation) and flips (reflections) on shapes
Year 3	moving, combining and drawing patterns of combinations of 2D shapes, and extend to moving 3-D objects in space
Year 4	knowing the spatial features that shapes and objects have and using these to sort and recognise shapes and objects in different places and orientations
Year 5	describing and modelling equivalence and difference in the context of shapes as well as numbers
Year 6	using formal language of geometrical properties to describe shapes, transformations and spatial positions
Year 7	using specific geometrical language that enables precision in describing and replicating shapes and design
Year 8	understanding features of shapes and space and moving between different models/representations of 2 and 3D shapes and objects
Year 9	using technology and geometrical relationships to resolve real world problems involving 2D shapes, scale and symmetry
Year 10	using technology, including geometry software, and geometrical relationships to understand and resolve real world problems involving 2D and 3D shapes, objects and symmetry
Year 11	selecting and using geometrical tools to investigate geometrical situations involving shape, position and symmetry across measurement, geometry and algebraic contexts

Statistics and Probability

	Main kaupapa/purpose/focus for ākonga
Two terms	sorting things into categories that show something about the collection of things
Year 1	using comparison to discuss investigations and their outcomes, including the language of chance
Year 2	working with others to pose a question, collect category data, sort it, discuss what was found and understand that not all responses or outcomes were equally likely
Year 3	working with more than one variable that describes a data set that they have collected and making sense of others' descriptions
Year 4	working with more than one variable that describes a data set collected by others and exploring the likelihood of particular outcomes
Year 5	thinking critically about category data displays created by themselves and others
Year 6	collecting and using time series data as well as summary and comparison data, and being able to model the likelihood of outcomes in controlled probability situations (such as rolling two dice)
Year 7	working with relationships in and through data, describing the shape of the data, and using continuous numerical variables
Year 8	engaging in all phases of a simple statistical investigation involving comparison or relationship situations, using and critiquing their own data sets and data sets from others, and be able to explain why the results of a probability experiment might not be the same as a model would predict
Year 9	understanding that variation and chance affect results of statistical investigations and probabilities of events happening
Year 10	understanding how the design of an experiment, data collection, sampling and analysis impact on findings of an investigation
Year 11	being aware of and able to critique chance and data situations relevant to important issues in their everyday lives and make decisions based on this awareness

Appendix Two: Example showing possible Important Cross-disciplinary links for mathematics and statistics with Te reo Māori

It is timely to ensure all ākonga have a growing vocabulary of kupu Māori for mathematics/pāngarau and tauanga/statistics and have opportunities to learn and use these terms and sentence structures in English and te reo Māori. Such knowledge and use assists ākonga to see purpose for and relevance of mathematics and statistics in their everyday lives and enables consideration of differing worldviews. For te reo Māori learners it would be ideal to have new mathematics and statistics focus te reo Māori learning for each year. It is suitable for kaiako to have guidance and support for this aspect of the teaching and learning of mathematics and statistics, including to help broaden and deepen the inclusion of te reo Māori within mathematics and statistics learning programmes.

The kupu Māori below are suggested as discussion starters regarding what makes for suitable knowledge of mathematical terms in te reo Māori for all ākonga in Aotearoa New Zealand across year levels as part of the key curriculum mathematical skills and knowledge. That is to say, they are included in this report as provocations for consideration rather than as a definitive selection. Consideration has been given to the Languages curriculum focus areas, mathematics content at the year levels, and potential cultural priorities. Consultation with suitable experts is expected.

It is assumed that language knowledge such as this will be developed cumulatively by kaiako with ākonga as ākonga progress through year levels.

Year	**Focus areas of the Languages curriculum[26] (levels 1-8)**		
	Language knowledge Knowing ngā kupu Māori to be able to:	**Communication** Sentence structures including and building on:	**Cultural knowledge (historical and current)**
0	communicate using counting numbers up to 20	e.g., Use numbers in simple sentences How many things? There are three things. How many people? There are four people.	Considering culturally located ways of thinking about quantity (how many) and that these can vary with worldview.
1	describe relationships between people	e.g., Use relationship words in sentences (e.g., mother, father, sister, brother, koro, kuia, tupuna, friend...) Who is your mother? Tere is my mother. How many sisters do you have? Who are your sisters?	Considering the personal relationships that are important to ākonga and their families and whānau.
2	describe position and following	e.g., Use position words in sentences (e.g., above, below, behind, in front of, on top of, under, beside...)	Considering culturally located ways of thinking about position and

[26] https://nzcurriculum.tki.org.nz/The-New-Zealand-Curriculum/Learning-languages/Achievement-objectives#collapsible1

	instructions related to position	Where is your bag? It is behind the chair. Stand beside the door.	instructions/commands and that these can vary with worldview.
3	describe the measure of an object and relative measures of objects	e.g., Use measurement words in sentences (e.g., long, short, hot, cold, longer, shorter, hotter, old, heavy, heaviest, big, small, smallest…) This is a long pencil. My cup is hotter.	Considering culturally located ways of thinking about quantity (measure) and that these can vary with worldview.
4	describe and communicate about simple shapes and spatial features	e.g., Use words for shapes in simple sentences (e.g., koru, patiki, circle, square, triangle, rectangle straight, angle…) What shape is the sandwich? The sandwich is a rectangle. These shapes have curves. Those shapes have straight edges.	Considering how features of shapes make shapes useful for particular purposes and explore how everyday objects utilise these properties.
5	describe and communicate about probabilities of events occurring	e.g., Use probability words in sentences (e.g., certain, never, likely, unlikely, 50:50, almost certain, impossible…) What are the chances of going to the shop today? Very likely. It is unlikely to rain tomorrow.	Considering likelihoods of events within culturally linked activities, e.g., in kapa haka competitions, powhiri, Iron Māori
6	communicate using names of simple fractions	e.g., Use fractions in simple sentences (e.g., falf, quarter, three quarters, a third, two thirds) I ate half the chocolate. What is the time? It is quarter past ten.	Considering culturally located ways of thinking about sharing.
7	describe and use compass directions	e.g., Use direction words in simple sentences (e.g., north, south, east, west…)	
8	…		
9			
10			
11			
12			
13			